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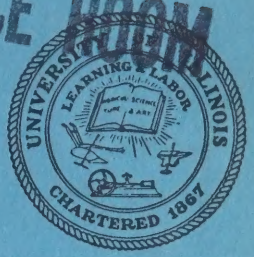

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FIFTH PROGRESS REPORT
INVESTIGATION OF PRESTRESSED CONCRETE FOR HIGHWAY BRIDGES

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OCTOBER, 1956

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FIFTH PROGRESS REPORT

OF THE

INVESTIGATION OF PRESTRESSED CONCRETE

FOR HIGHWAY BRIDGES

Conducted by

THE ENGINEERING EXPERIMENT STATION

UNIVERSITY OF ILLINOIS

In Cooperation with

THE DIVISION OF HIGHWAYS

STATE OF ILLINOIS

and

THE U. S. DEPARTMENT OF COMMERCE

BUREAU OF PUBLIC ROADS

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SUMMARY OF ACTIVITIES

1. History of Project

This investigation of prestressed concrete for highway bridges was begun in the summer of 1951 as a cooperative project of the University of Illinois Engineering Experiment Station and the Illinois Division of Highways, with participation by the U. S. Department of Commerce, Bureau of Public Roads through Federal Aid.

Four previous progress reports have covered the work done up to September 1955. This report summarizes the activities on the project from September 1955 to date.

2. Organization

The work on this investigation has been carried out in the Structural Research Laboratory of the Department of Civil Engineering under the general supervision of N. M. Newmark, Research Professor of Structural Engineering; C. P. Siess, Research Professor of Civil Engineering; and N. Khachaturian, Assistant Professor of Civil Engineering. Guidance to the project staff has been provided by an Advisory Committee including representatives from each of the cooperating agencies. During the past year, the membership of the Advisory Committee has been as follows:

Representing the Illinois Division of Highways:

W. E. Chastain, Sr., Engineer of Physical Research,
Bureau of Research and Planning

W. J. MacKay, Bridges and Traffic Structures Section, Bureau of Design

C. E. Thunman, Bridges and Traffic Structures Section, Bureau of Design

Representing the Bureau of Public Roads:

E. L. Erickson, Chief, Bridge Branch

E. F. Kelley, Chief, Physical Research Branch

Representing the University of Illinois:

C. P. Siess, Chairman

I. M. Viest, Research Associate Professor of Theoretical and Applied Mechanics

N. Khachaturian, Assistant Professor of Civil Engineering

The work on the project has been under the immediate supervision of M. A. Sozen, Research Associate in Civil Engineering. The remaining project personnel are all graduate students in Civil Engineering, devoting from one-half to full time to research on this project. They are: J. Alagia, I. Ali, R. J. Allen, O. R. Gardi, P. Murphy, G. Hernandez, and J. Warwaruk.

3. Scope of Work

The work now underway on the project can be divided into four categories:

(a) Experimental studies of beams failing in flexure.

(b) Analytical studies of beams failing in flexure.

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(c) Studies of beams failing in shear.

(d) Studies of time-dependent effects.

In the following sections, brief outlines of the work carried out in these four categories are presented.

4. Experimental Studies of Beams Failing in Flexure

Mr. R. J. Allen, with the assistance of Mr. J. Warwaruk, carried out the work on this phase of the project. Mr. Allen's thesis, representing the analysis of the results of tests on 30 simply-supported unbonded beams, has been reproduced as Structural Research Series No. 119 and distributed as a part of this Fifth Progress Report.

In a thesis published as a part of the Third Progress Report of this investigation, Mr. Feldman presented the results of tests on 16 unbonded beams, and showed that such specimens have lower strengths than comparable bonded beams, and that the ultimate strength of unbonded beams may be estimated by recognizing the non-linearity in the strain distribution over the depth of the beam. Mr. Allen's tests were aimed at establishing the relation between this strain distortion and several critical parameters which were expected to affect it.

At the conclusion of Mr. Allen's tests the following variables had been covered:

(1) Concrete strength,

(2) Percentage of unbonded high-tensile-strength reinforcement,

(3) Bonded intermediate grade steel reinforcement, and

(4) Ratio of flexure span to total span.

The effective prestress, cross-sectional dimensions, and total span length were held constant at values of 120,000 psi, 6 x 12 in., and 9 ft. respectively.

Studies of the distortion factor, F, with respect to the variables considered revealed that it was affected by the depth of the compression zone at failure which in turn was a function of the ratio of the amount of reinforcement to the concrete strength, and the number of cracks in the beam, this number being governed by the length of flexure span. It was also found that the addition of mild steel reinforcement at the bottom of the unbonded beam did not improve the strength perceptibly, although it did cause a more uniform distribution of cracks.

In general, these tests have shown that the strength of an unbonded prestressed concrete beam is inferior to that of a bonded beam, and that this difference increases with the ratio of the percentage of longitudinal steel to the concrete strength, and decreases as the length of the flexure span or the level of prestress is increased. One salient feature that always differentiated an unbonded beam from a bonded beam was the state of the beam after failure. In all cases except where mild steel bonded reinforcement was used, total destruction of the beam occurred at the ultimate load as compared to the more or less ductile type of failure exhibited by bonded beams.

5. Analytical Studies of Beams Failing in Flexure

The analytical studies of design criteria for prestressed concrete beams were carried out by Mr. J. Alagia and Mr. I. Ali under the direction of Professor Khachaturian. Mr. Alagia worked on this phase of the investigation during the fall semester of 1955. Mr. Ali continued the work during the spring semester of 1956 and the following summer. A report describing these analyses is being prepared and will be issued separately during the coming year.

These studies have been generally similar to those reported previously for rectangular sections* except that they have dealt with the more typical and much more complex problem of the unsymmetrical I or T-shaped cross-section.

Algebraic expressions have been derived in terms of dimensionless combinations of variables to relate the several conditions governing design at working loads. With the aid of these expressions, criteria have been developed for "least weight" designs which, for a given set of allowable stresses and assumed loss in prestress, can be represented graphically in such a manner as to permit the design of unsymmetrical cross-sections for least weight.

Simple expressions have also been developed for the total load factor of safety, assuming failure in flexure, in terms of

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* F. R. Khan, N. Khachaturian, and C. P. Siess, "Analytical Studies of the Relations Among Various Design Criteria for Prestressed Concrete Beams," issued as a part of the Fourth Progress Report of Prestressed Concrete for Highway Bridges, Civil Engineering Studies, Structural Research Series No. 105, October 1955.

the same dimensionless parameters considered in design at working loads. By means of curves based on these two sets of expressions for working load and ultimate, it is possible to determine the total load factor of safety for "least weight" designs corresponding to any given set of specified stresses and prestress losses. In addition, curves have been constructed to relate total load and live load factors of safety.

Except for the assumption that the beams are under-reinforced and fail in flexure, the expressions which have been derived are entirely general and apply to beams of any shape or cross-section and to any set of allowable stresses, prestress losses, etc. Although the current studies have been limited to specific criteria and design specifications, they may be extended to cover other cases and to investigate other existing or proposed criteria to establish the relationships between working loads and ultimate strengths.

6. Studies of Beams Failing in Shear

Analytical and experimental work in connection with the shear strength of prestressed concrete beams was carried out by Mr. M. A. Sozen with the assistance of Mr. G. Hernandez. A brief description of this work is contained in Part 2 of this volume.

The purpose of this phase of the investigation is to determine the strength and modes of failure of simply-supported prestressed concrete beams failing in shear. With the testing of 17 I-beams during the past year, the total number of specimens tested in connection with

these shear studies rose to 104 beams without web reinforcement, including rectangular and I-beams, all but a few of which failed as a result of inclined cracking.

The main variables in these tests were:

(1) concrete strength,

(2) amount of longitudinal steel,

(3) level of effective prestress,

(4) ratio of web to flange thickness,

(5) ratio of shear span to effective depth,

(6) length of span.

Studies of the available data indicated that a fully-developed inclined crack is a necessary and sufficient cause of a shear-initiated failure as differentiated from a flexural failure by the reduction in the ultimate steel strain. It was also observed that at practical levels of prestress the additional load capacity of the simply-supported prestressed beam after the formation of the fully-developed inclined crack was comparatively small. The qualitative deductions regarding the behavior of beams failing in shear, and some of the quantitative implications of these tests, are discussed in Part 2 of this volume. The data and the conclusions will be completely presented in a Ph.D. thesis by M. A. Sozen to be distributed sometime during 1956-57.

7. Studies of Time-Dependent Variables

Mr. O. R. Gardi and Mr. P. E. Murphy worked on the study of time-dependent effects in prestressed concrete during the first

and second semesters of the school year 1955-56, respectively. Mr. Gardi's studies on the combined effects of time-dependent variables in prestressed concrete beams were extended by Mr. T. J. Larsen, senior student in Civil Engineering, and are presented in Part 3 of this volume.

(a) Study of Relaxation Properties of High-Tensile Strength Steel. The experimental study of relaxation properties of prestressing wire was continued within the past year. Accumulated data included readings at 22,000 hours at the time of issue of this report. Tests have been conducted on 46 specimens. Of these, 16 are still under test. The following variables were covered in these tests:

(1) source of wire

(2) type of wire

(3) initial stress level

(4) effect of prestretching

Four of the specimens represented wire manufactured by the Union Wire Rope Corporation of Kansas City, Missouri. Two tests were made on a special type of wire obtained from the Somerset Steel Company of England. The remaining 40 specimens were cut from different bundles of wire manufactured by the American Steel and Wire Division of the United States Steel Corporation.

Six of the specimens were straightened but not stress-relieved. Eight of them were straightened and stress-relieved. Thirty were not straightened but stress-relieved. The two specimens of Somerset wire had received a special heat treatment under stress during manufacture.

The initial stress level varied from 101,000 to 213,000 psi. Represented as a percentage of the ultimate stress, the variation in initial stress level was from 51 to 89 per cent.

Sixteen specimens were prestretched by stressing the wire to about 115 per cent of the intended initial stress and holding it at that level for about 10 to 15 minutes.

(b) Combined Effects of Time-Dependent Variables in Prestressed Concrete Beams. The object of this investigation is to study the combined effects of creep and shrinkage in the concrete and relaxation of the steel on the time-behavior of prestressed concrete members. Time-deflection curves obtained from two loaded and two unloaded prestressed concrete beams which have been under test for a period of 15,000 hours were compared with curves computed on the basis of several simplifying assumptions. These curves are presented in Part 3 of this volume.

8. Future Work

The work now under way or planned for the year 1956-57 includes primarily a continuation of the several studies now constituting the comprehensive program, as follows:

(a) Experimental Studies of Beams Failing in Flexure. Tests will be made on pretensioned beams to investigate conditions intermediate between the fully bonded and the fully unbonded cases previously studied. Tests may also be made on beams with unbonded prestressed reinforcement with supplementary bonded but not prestressed reinforcement consisting of high-tensile-strength single wires.

(b) Analytical Studies of Beams Failing in Flexure. The study of interrelationships between working load and ultimate load designs for unsymmetrical sections will be extended and related to practical design limits in an attempt to determine minimum values of factors of safety corresponding to the various possible working load design criteria.

(c) Shear. Tests will be made on pretensioned I-beams with web reinforcement consisting of mild steel vertical stirrups, to determine the minimum amount and spacing of stirrups required to prevent failure in shear.

(d) Time-Effects. Relaxation tests on high-tensile-strength wires will be continued, with emphasis on relaxation losses at high stresses. New sustained load tests on prestressed concrete beams will be started. Long time behavior studies will be made of typical prestressed concrete bridges on the basis of the implications of the sustained load test results reported in this volume.

9. Reports and Publications

The reports and publications issued during the course of this investigation are listed in chronological order.

"First Progress Report - Investigation of Prestressed Concrete for Highway Bridges," Engineering Experiment Station, Structural Research Series No. S-9, April, 1952.

"Bibliography on Prestressed Concrete," Engineering Experiment Station, Structural Research Series No. S-10, April, 1952.

"Tests of a Full-Size Prestressed Concrete Block Beam," Reported by J. H. Appleton, Conducted as a Part of the Investigation of Prestressed Concrete for Highway Bridges, June, 1952.

"Shear Strength of Simply-Supported Prestressed Concrete Beams," a Thesis by E. M. Zwoyer, Issued as a Part of the Second Progress Report of the Investigation of Prestressed Concrete for Highway Bridges, Civil Engineering Studies, Structural Research Series No. 53, June, 1953.

"Study of Prestressed Concrete Beams Failing in Flexure," a Thesis by D. F. Billet, Issued as a Part of the Second Progress Report of the Investigation of Prestressed Concrete for Highway Bridges, Civil Engineering Studies, Structural Research Series No. 54, June, 1953.

"Second Progress Report - Investigation of Prestressed Concrete for Highway Bridges," June, 1953.

"A Study of Time-Dependent Variables in Prestressed Concrete," a Thesis by C. McLean, Issued as a Part of the Third Progress Report of the Investigation of Prestressed Concrete for Highway Bridges, Civil Engineering Studies, Structural Research Series No. 74, June, 1954.

"Bonded and Unbonded Prestressed Concrete Beams Failing in Flexure," a Thesis by A. Feldman, Issued as a Part of the Third Progress Report of the Investigation of Prestressed Concrete for Highway Bridges, Civil Engineering Studies, Structural Research Series No. 75, June, 1954.

"Third Progress Report - Investigation of Prestressed Concrete for Highway Bridges," June, 1954.

"Flexural Strength of Prestressed Concrete Beams," D. F. Billet and J. H. Appleton, Journal of the American Concrete Institute, V. 25, No. 10, June, 1954 (Reprint issued as Structural Research Series No. 85)

"The Ultimate Strength in Shear of Simply-Supported Prestressed Concrete Beams Without Web Reinforcement," E. M. Zwoyer and C. P. Siess, Journal of the American Concrete Institute, V. 26, No. 2, October, 1954. (Reprint issued as Structural Research Series No. 85)

"Analytical Studies of the Relations Among Various Design Criteria for Prestressed Concrete Beams," F. R. Khan, N. Khachaturian, and C. P. Siess, Issued as a Part of the Fourth Progress Report of the Investigation of Prestressed Concrete for Highway Bridges, Civil Engineering Studies, Structural Research Series No. 105, October, 1955.

"Fourth Progress Report - Investigation of Prestressed Concrete for Highway Bridges," September, 1954.

* "Relaxation of High-Tensile-Strength Steel Wire for Use in Prestressed Concrete," C. P. Siess and G. McLean, American Society for Testing Materials Bulletin No. 211, January, 1956. (Reprint Issued as Structural Research Series No. 117.)

* "Strength of Prestressed Concrete Members," C. P. Siess, Symposium on the Strength of Concrete Structures, Session B, Paper No. 3, Cement and Concrete Association, London, May, 1956.

* "Flexural Strength of Unbonded Prestressed Concrete Beams," a Thesis by R. J. Allen, Issued as a Part of the Fifth Progress Report of the Investigation of Prestressed Concrete for Highway Bridges, Civil Engineering Studies, Structural Research Series No. 119. June, 1956.

* "Fifth Progress Report - Investigation of Prestressed Concrete for Highway Bridges," October, 1956.

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* Issued since Fourth Progress Report, September, 1955.

PART 2

PROGRESS REPORT

on

STUDIES OF BEAMS FAILING IN SHEAR

by

M. A. Sozen

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I. INTRODUCTION

1. Object

Since this investigation of prestressed concrete for highway bridges began in 1951, 104 beams without web reinforcement and four beams with web reinforcement have been tested in connection with the study of simply-supported prestressed concrete beams failing in shear. This report is intended: (a) to describe briefly the series of 17 beams tested within the past year, (b) to present an outline of the entire series of tests completed to date, (c) and to indicate some of the more significant conclusions drawn from the tests of beams without web reinforcement which are to be presented in detail in a pending report.

2. Scope

The ultimate aim of the present phase of this investigation is the establishment of generally applicable criteria for the design of prestressed concrete beams against shear-initiated failures. This report considers the results of the tests in relation to one such criterion, the load producing the first major inclined crack. The modes of failure following the initiation and full development of an inclined crack are mentioned only briefly in this report and are to be discussed more fully in a thesis to be presented in the near future.

The following variables have been considered to date:

(1) concrete strength

(2) amount of longitudinal reinforcement

(3) level of effective prestress

(4) ratio of shear-span to effective depth

(5) ratio of web to flange thickness

(6) length of span

The four rectangular beams with web reinforcement were tested as an exploratory series to study the effect of web reinforcement on shear strength. The only variables in these tests were the amount of web reinforcement, the concrete strength, and the level of effective prestress.

3. Outline of Tests

The properties of the 17 I-beams V-47 through V-63, which were tested during the past year, are presented in Table I of this report. Corresponding data for the remainder of the test specimens have been given in previous reports, as follows:

Beams S-1 through S-38: Thesis by E. Zwoyer published as a part of the Second Progress Report.

Beams V-1 through V-30: Third Progress Report.

Beams V-31 through V-46a and R-1 through R-6: Fourth Progress Report.

The overall dimensions for all of the 108 test beams were 6 x 12 in. in cross-section. The nominal dimensions for the I-beams are given in Fig. 1a and 1b of the Third Progress Report. All tests

were conducted on simply-supported specimens loaded symmetrically about mid-span or at mid-span. Except for four rectangular beams which were tested on a span of 7 ft, the span was kept constant at a length of 9 ft. All beams had straight tension reinforcement and no compression steel. The reinforcing steel came from six different lots of Supertens prestressing wire manufactured by the American Steel and Wire Division of the United States Steel Corporation. The characteristics of the wire varied slightly from lot to lot. The maximum and minimum diameters recorded were 0.192 and 0.196 in.

Thirty-four rectangular beams of the S-series were post-tensioned. All other test specimens were pretensioned.

Of the 108 test specimens, 104 had no web reinforcement. The four beams with web reinforcement, tested as an exploratory series, will not be discussed at this time. The specimens without web reinforcement were divided into the following groups with respect to their cross-section: (a) 40 rectangular beams, (b) 41 I-beams with nominal web thicknesses of 3 in., and (c) 23 I-beams with nominal web thicknesses of 1 3/4 in.

The level of effective prestress varied from zero to a maximum of 140,000 psi. The highest level of prestress corresponded to about 60 per cent of the ultimate strength of the prestressing wire. Of the 40 rectangular beams, nine had effective prestresses in the range from zero to 30,000 psi; 12 were prestressed to levels ranging between 60,000 and 80,000 psi; and there were 19 beams with prestresses in the range from 100,000 to 140,000 psi. The I-beams were pretensioned

to nominal stresses of 0, 60,000, and 120,000 psi. Six of the I-beams with 1 3/4-in. webs were not prestressed, eight were stressed to 60,000 psi, and nine were stressed to 120,000 psi, nominally. The I-beams with 3-in. webs can be classified according to the nominal effective prestress level as follows: nine at zero, ten at 60,000, and two at 120,000 psi prestress.

The upper and lower limits for the ratio of the shear span to the effective depth were 6.7 and 2.8, respectively. Four rectangular beams in the 60,000 to 80,000-psi prestress group were tested with a shear span of 24 in. Three from the 60,000-psi prestress group and four from the 100,000 to 140,000-psi prestress group were tested at a shear span of 54 in. There was no variation in the shear span of 36 in. for the I-beams with 1 3/4-in. web thickness. Six 3-in. web I-beams were tested with a shear span of 54 in., and four with a shear span of 28 in. All but two of these specimens belonged to the 120,000 psi prestress group. One of the two, both of which had shear spans of 54 in., was not prestressed while the other had a nominal prestress of 60,000 psi.

The concrete strength varied from 1750 psi to 8600 psi as measured from 6 by 12-in. cylinders. Because of difficulties caused by the fragility of the I-beams within the first few days after casting and the necessity of removing the side forms to prevent sticking within the same period, trouble was encountered in the manufacture of beams with concrete strengths lower than 3500 psi. However, it is hoped that in the future, as in the last four specimens cast, these

difficulties will be avoided by pulling the side forms away within three or four hours after casting rather than the previously followed practice of waiting for at least 24 hours to carry out this function.

The percentage of longitudinal steel ranged between 0.179 and 0.877 per cent. In beams with relatively high percentages of reinforcement, the excessive tension at the top fiber was overcome by the application of an external prestress to the top flange before release of the wires. This external prestress was released in the early stages of the test.

It should be noted here that all but three of the I-beams were tested with externally applied prestressed stirrups placed on each side of each bearing plate at the supports.

4. Notation

The following notation is used in this report:

A_c = cross-sectional area of concrete

A_s = cross-sectional area of steel

a = length of shear-span

b = width of flange

b' = width of web

d = effective depth of the beam

E_s = Young's modulus for steel

F = effective prestressing force

f_{se} = effective prestress

f'_c = concrete strength

f_r = modulus of rupture of concrete

f_t = tensile strength of concrete

M_u = ultimate moment

M_c = moment at inclined tension cracking

P_c = load at inclined tension cracking

P_u = ultimate load

p = ratio of cross-sectional area of steel to that of concrete

$Q = \frac{pE_s}{f'_c}$

II. DISCUSSION OF TEST RESULTS

5. Inclined Tension Cracking

(a) Effects of Inclined and Horizontal Tension Stresses. In order to satisfy equilibrium conditions, tensile stresses develop in a transversely loaded structural member acting essentially as a beam. These stresses are horizontal, that is, parallel to the longitudinal axis of the beam, at points where no shearing stresses exist. At points where there are shearing stresses, the maximum principal tensile stress is inclined with respect to the longitudinal axis of the beam. In members made of isotropic materials, the tensile stresses are usually of no greater consequence than compressive stresses; however, for those made of concrete, for which tensile and compressive strengths are normally in the ratio of 1 to 12, the tensile stresses are critical.

In order to make full use of the compressive strength of concrete, it has become common practice always to use horizontal steel reinforcement on the tension side of concrete beams. It has not been common practice, however, always to provide reinforcement against the inclined tensile stresses. Web reinforcement is frequently omitted, sometimes with and sometimes without good reasons.

The following discussion of the cracking of concrete under tension is confined to simply-supported bonded prestressed concrete beams without web reinforcement, loaded statically over moderate spans of constant or nearly constant shear. It is assumed that

cracking of the concrete is a stress phenomenon, and its very probable time-dependence is ignored.

As the load on the beam is increased, a crack caused by the horizontal tensile stress is initiated at the bottom fiber in a region of maximum bending moment when the tensile stress exceeds the sum of the compressive stress on that fiber and the modulus of rupture, this latter quantity being determined with the aid of the conventional flexure formula from plain concrete beams stressed more or less in the same manner. This crack is vertical.

An inclined-tension crack starts at a point in the shear span when the sum of a component of the compressive stress in a certain direction and the tensile strength of the concrete is exceeded by the tensile stress in that direction. The tensile strength of the concrete in the beam is not as easy to determine as the modulus of rupture. The inclined crack may be initiated either prior or subsequent to a vertical crack at the same location and with almost any inclination, depending on the combination of shear, flexure, and prestressing stresses.

As the horizontal-tension crack moves perpendicularly towards the neutral axis of the beam, more and more of the necessary internal tensile reaction is supplied by the horizontal steel.

The propagation of the inclined-tension crack is accompanied by an increase in the inclined tensile stresses which is only partially compensated by the horizontal reinforcement.

The penetration of the horizontal-tension crack is limited by the relative load-carrying capacities of the horizontal steel in tension and the concrete in compression. The crack extends almost to the neutral axis.

The propagation of the inclined-tension crack is never stopped, but only diverted by the relatively high compressive stresses in the compression zone of the beam.

The horizontal-tension crack does not greatly disturb the linearity of the strain distribution over the depth of the cross-section. Beam-action is preserved at all stages of its development.

The inclined-tension crack distorts the strain distribution over the depth of the cross-section severely. After its full development, beam-action is replaced by arch-action, and the member may fail in a manner associated with the latter. Following the formation of the inclined-tension crack, bond and horizontal shear stresses assume greater significance.

If full use is to be made of the carrying capacity of a beam that can develop inclined cracking, then omitting the web reinforcement is no more logical than omitting the longitudinal reinforcement in a beam prestressed by external means (jacks).

If the design capacity of a beam is to be related to its ultimate capacity, and if this beam has the propensity to develop inclined cracks, reinforcement against inclined tension is imperative. Once this is resolved, two questions of a quantitative nature arise. These are, "when" and "how". Difficulties involved in answering the

first inquiry have been implied in the preceding paragraphs, and will be discussed in connection with the experimental results later in this section. In theory, the answer to the second question is relatively simple. In practice, it involves problems of economy and has not been adequately investigated, at least for prestressed concrete beams. Naturally, both these questions are ruled out if design philosophy tolerates shear-initiated failures.

(b) Experimental Data on Inclined Tension Cracking. In compiling the test data on inclined tension cracking, the inclined-tension cracking load was regarded as the applied load at which the crack first showed signs of "flattening out". Little controversy was introduced by this definition, because, especially in prestressed beams, once the inclined tension crack was initiated, it attained its maximum height with little increase in applied load. The inclined tension crack was observed to intersect the longitudinal reinforcement at about a distance equal to the effective depth away from the point of application of the load. This distance increased with increase in shear span. Although horizontal-tension cracking almost always preceded inclined tension cracking in the shear span in the rectangular specimens, in the I-beams with high amounts of longitudinal steel and prestress, horizontal-tension cracking was preceded by inclined tension cracking. The inclination of the initial inclined tension crack was in general flatter for beams with higher effective prestressing forces. This inclination also appeared to decrease with increase in the shear

span. The various anomalies and inconsistencies in inclined tension cracking are described in Section 12, Part 3, of the Fourth Progress Report.

The tensile strength of the concrete, as stated earlier, contributes to the capacity of the beam at inclined tension cracking; the relative importance of this contribution decreases as the ratio of the prestressing force to the cross-sectional area increases. Since no tests giving the tensile strength of the concrete directly were available, this quantity was assumed to be 2/3 of the modulus of rupture as determined from tests on 6 by 6 by 24 in. plain concrete beams, loaded at the third-points on an 18-in. span. The proportion 2/3 was based upon the extreme assumption that at the time of rupture of these control beams the stress distribution was rectangular.

The relation between the modulus of rupture and compressive strength of concrete has been found to be modified by the size of the coarse aggregate. Since, for convenience in casting, coarse aggregate with its largest dimension less than 3/8 of an inch was used in pre-tensioned beams, as compared to aggregate with normal gradation in the post-tensioned specimens, two expressions were derived for the modulus of rupture. The data pertaining to beams with regular and small size coarse aggregate are shown in Figs. 1 and 2, respectively. The empirical expressions selected to represent this type of data are shown below and also on the pertinent figures.

For concrete with regular coarse aggregate:

$$f_r = \frac{3000}{2 + \frac{12000}{f'_c}} \; ; \quad f_t = \frac{2000}{2 + \frac{12000}{f'_c}}$$

For concrete with small size coarse aggregate:

$$f_r = \frac{3000}{4 + \frac{12000}{f'_c}} \; ; \quad f_t = \frac{2000}{4 + \frac{12000}{f'_c}}$$

The values for the modulus of rupture, f_r, the tensile strength, f_t, and compressive strength, f'_c, are all in pounds per square inch.

The data on inclined tension cracking were expressed as functions of various properties contributing to the strength of the beam, such as the tensile strength and the geometrical properties, and compared with various parameters involving factors which affect the tensile stresses in the beam. The most consistent and significant relation was found to be the one between $M_c/f_t bd^2 \sqrt{b'/b}$ and $F/A_c f_t$ as shown in Fig. 3. It should be repeated here that this plot includes data from tests with three different web thicknesses and with three different lengths of shear span. The abscissas of this plot are the ratios of the average compressive "prestress" to the tensile strength, and would be the rational parameter to use had the inclined tension crack started at the neutral axis of an uncracked section. The form-factor is partially reflected in the denominator of the ordinates by the term $\sqrt{b'/b}$. The variations in the ratio of the shear span to the effective depth were observed, at least for the range of these tests, to affect the nominal shear at inclined tension cracking, $\frac{V}{bd}$,

directly. Hence, the ordinates are expressed in terms of the moment. Since the tensile strength of the concrete, f_t, is the denominator of both the ordinates and the abscissas, it is seen that the inaccuracies in predicting this value are mostly cancelled out.

The following empirical expression was selected to describe the relationship indicated by the data:

$$\frac{M_c}{f_t b d^2 \sqrt{\frac{b'}{b}}} = 1 + \frac{F}{A_c f_t}$$

Exploratory theoretical studies of inclined tension stresses in prestressed concrete beams with cracked sections indicate that the close agreement of the experimental results may be partially the reflection of the fact that when a beam is loaded there is a value of the external shear (in the vicinity of the observed horizontal-tension cracking shear) at which the inclined tension stresses increase at a very fast rate with increase in the external shear. Evidently, although the governing tensile stress may vary within quite an extensive range, the measured value of the external shearing force will not be subject to great scatter. Because of this relation between the inclined tensile stresses and the applied shear, it is felt that, although the test results pertain to specimens symmetrically shaped about the mid-height, the empirical expression should be reliable also for T-beams.

It should be briefly noted here that in prestressed concrete beams with relatively high values of the parameter $F/A_c f_t$, the maximum

tensile stresses may occur in the upper portion of the beam at locations near the ends of the span where the bending stresses are small. Thus the initial tension crack can start above mid-depth of the beam. This type of crack has not been critical in most of the tests reported here, since the I-beams had end-blocks and prestressed stirrups at each end which alleviated such stresses. However, such cracks have been observed in several I-beams with high values of the parameter $F/A_c f_t$. Since the formation of these cracks was affected by such special conditions as the size of the end-block and the location and level of prestress of the externally applied stirrups, no attempt was made to investigate this phase of the problem on the basis of the empirical data.

6. Behavior of Prestressed Concrete Beams

The following discussion of the behavior of prestressed concrete beams is undertaken in an effort to describe qualitatively the modes of failure observed in the course of the tests reported in this volume. Consequently, it applies strictly to simply-supported, bonded beams without web reinforcement, loaded statically at one or two points along the span. However, its implications should carry over to more complicated structures where simple parts under comparable conditions can be isolated from the rest.

If bond distress prior to the formation of inclined tension cracks is regarded as a result of poor proportioning and therefore non-typical, failures of prestressed concrete beams can be classified

primarily as flexural failures and shear failures. Insofar as the carrying capacity and the manner of failure is concerned, there is no clear cut transition point between these two types of failure in terms of any one of the variables affecting the strength of prestressed concrete beams. However, if shear-initiated failures are defined as below, then, theoretically at least, values differentiating between flexural and shear failures can be assigned to the pertinent parameters.

A beam is said to have failed in flexure when its collapse is initiated by fracture or yielding of the longitudinal reinforcement, or by crushing of the concrete above a horizontal tension crack. This type of failure, at practical ratios of longitudinal steel to concrete strength, is not violent and gives ample warning of its approach. After failure the beam is not totally destroyed.

A beam is said to have failed in shear if its collapse is initiated by an inclined tension crack. Except for cases where the flexural capacity is only little higher than the inclined tension cracking load, this type of failure is violent, and usually occurs with little warning, if any. After failure the beam is totally destroyed.

Thus, the inclined tension crack is a necessary and sufficient condition for shear failures, although in the range of transition from flexural to shear failure the only effect of such a crack would be to reduce the ultimate steel strain.

The least violent mode of shear failure is the shear-compression failure. This type of failure is said to occur when a beam fails by crushing of the concrete at a steel strain less than

that which would have been developed for a flexural failure. The strain distortion over the depth of the section is introduced by the inclined tension crack which effectively "unbonds" the longitudinal steel.

For beams with webs thinner than the flange, the development of an inclined tension crack, which transforms the beam partially or completely into an arch, may be followed by crushing of the concrete in the web at a location along the path of the thrust. This type of failure is designated as a "web crushing" failure.

The length in which the tensile force in the steel is transferred to the compression side of the beam is cut down by the inclined tension crack. Therefore, failure may be caused by bond distress between the steel and the concrete, and, in thin-webbed beams, by separation of the tension flange from the web.

The properties which govern the mode of failure have been observed to be the following: (1) concrete strength, (2) amount of longitudinal reinforcement, (3) level of effective prestress, and (4) ratio of web to flange thickness. In general, it can be said that an increase in the amount of longitudinal reinforcement induces a shear failure, while increases in the rest of the listed properties induce a flexural failure. The particular values of the various parameters at which the type of failure changes from flexure to shear can be obtained from a comparison of the empirical expression presented in this report for the inclined tension cracking load with the flexural theory. The transition between the final modes of shear

failures assumes significance only if the additional load capacity between inclined tension cracking and collapse is to be considered for design purposes.

In Fig. 4, the ratios of the measured ultimate load to the inclined tension cracking load for all the beams failing in shear are shown plotted against their respective values of Q. The choice of Q for the abscissa is incidental since the plotted ratio is also a function of other variables. The data pertaining to beams at different levels of prestress and with different web thicknesses are shown by separate symbols.

The majority of the points representing beams at practical levels of prestress, that is beams with prestress close to or greater than 120,000 psi, fall within 25 per cent of the computed inclined tension cracking load. The ratios of the ultimate to the inclined tension cracking load are highest for beams with zero prestress. This observed trend is evident in the relations of the flexural capacity and inclined tension cracking load to the level of prestress. The flexural capacity is relatively insensitive to variations in prestress, while the inclined tension cracking load appears to be almost directly dependent on it. Also, at constant prestress, the ratio in question approaches unity with increase in Q, and with decrease in web thickness.

For beams with normal prestresses, the observed 25 per cent or less increase in load after inclined tension cracking can be explained and estimated semi-rationally when the failure is by shear-compression.

Data obtained in the reported tests have shed some light on the likelihood and effects of failure by crushing of the web or by separation of the tension flange from the web. It should be noted here that although indications of the initiation of this latter type of failure were observed in the tests, the final collapse of the beam in such cases was always due to crushing of the web since the end-block and the external stirrups on the end-block prevented a total separation of the flange and web. The presence of the end-block also influenced the web-crushing load, thus limiting the applicability of empirical conclusions drawn from these tests. However, one significant observation which is rationally justifiable is that for these two modes of failure the ultimate and inclined tension cracking loads are very close.

It appears from the trend of the data obtained to date that the inclined tension cracking load is a very significant point in the loading history of a beam. When it is considered that after the formation of such a crack the behavior of the beam becomes quite unstable and failure follows shortly and violently, it seems reasonable to base the factor of safety for beams without web reinforcement, if such beams are to be allowed at all, on the inclined tension cracking load in all cases where this load is less than the flexural capacity.

TABLE 1

PROPERTIES OF TEST SPECIMENS

Mark	Concrete Strength f'_c psi	Flange Width b in.	Web Thickness b' in.	Effective Depth d in.	Area of Steel A_s sq. in.	Longit. Reinf. p %	Effective Prestress f_{se} psi	Shear Span a in.
V47	3275	6.00	2.96	10.59	0.299	0.444	0	36
V48	3160	6.05	1.79	10.11	0.299	0.489	57.7	36
V49	3300	6.00	3.00	10.01	0.299	0.498	115.8	36
V50	3000	6.00	1.81	10.00	0.478	0.797	0	36
V51	3160	6.00	3.03	9.98	0.299	0.499	58.0	36
V52	3020	6.00	1.80	10.00	0.299	0.498	116.7	36
V53	3200	6.00	2.98	10.38	0.358	0.576	0	36
V54	2910	6.00	1.75	9.91	0.419	0.704	55.3	36
V55	2780	6.00	3.00	9.92	0.359	0.602	116.0	36
V56	7310	6.14	1.77	11.06	0.179	0.264	0	36
V57	3330	6.00	3.00	9.90	0.419	0.705	56.7	36
V58	3100	6.00	1.83	9.91	0.359	0.594	117.0	36
V59	5540	6.00	3.00	10.04	0.179	0.297	128.5	28
V60	4600	6.00	2.94	10.03	0.239	0.397	124.0	28
V61	4320	6.00	2.90	10.04	0.359	0.596	118.5	28
V62	4190	5.95	2.95	10.00	0.239	0.401	124.0	54
V63	4500	5.95	2.95	10.00	0.359	0.603	117.0	54

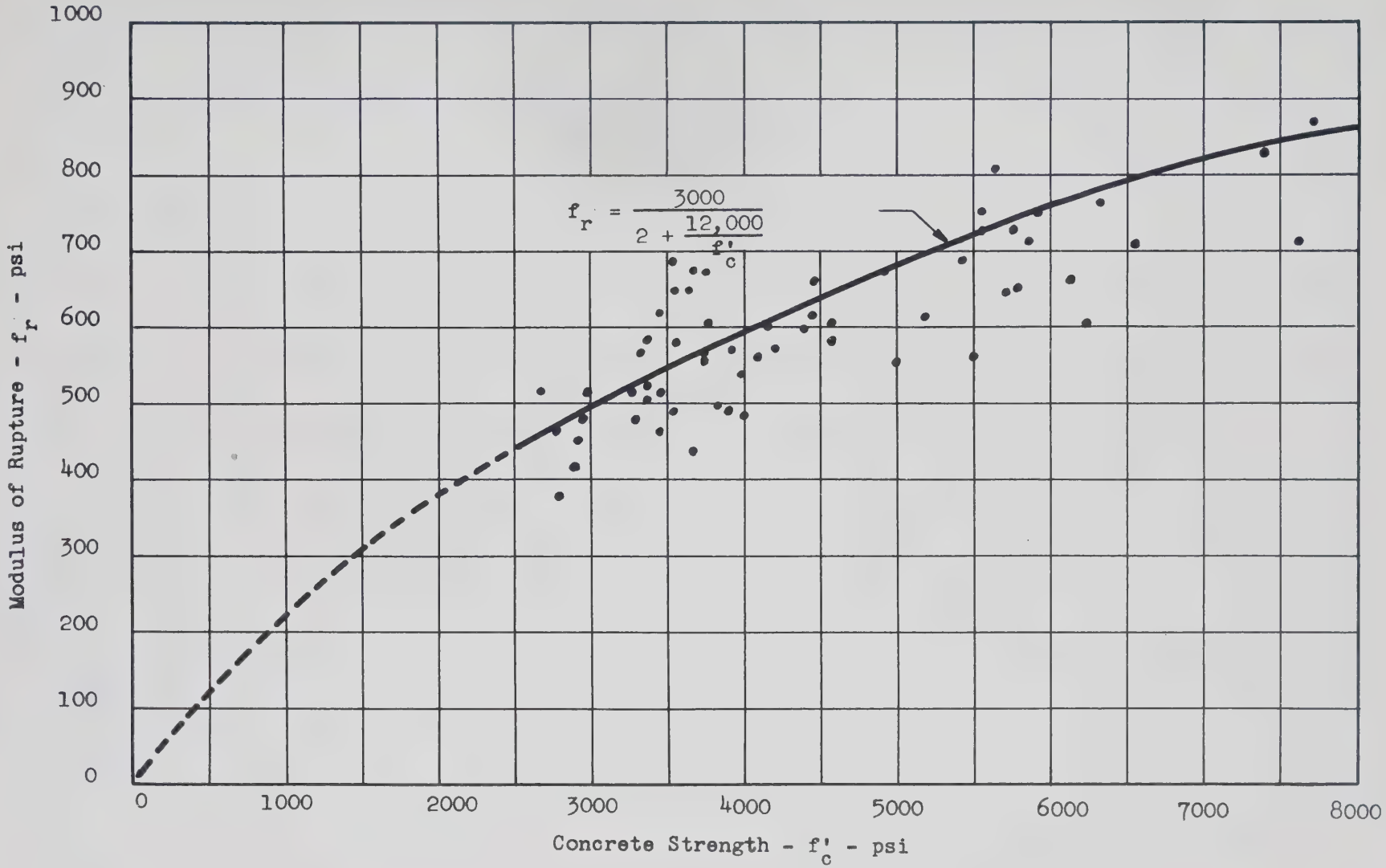

FIG. 1 COMPARISON OF MODULUS OF RUPTURE WITH CONCRETE STRENGTH
MAXIMUM SIZE OF COARSE AGGREGATE: 1-1/2 in.

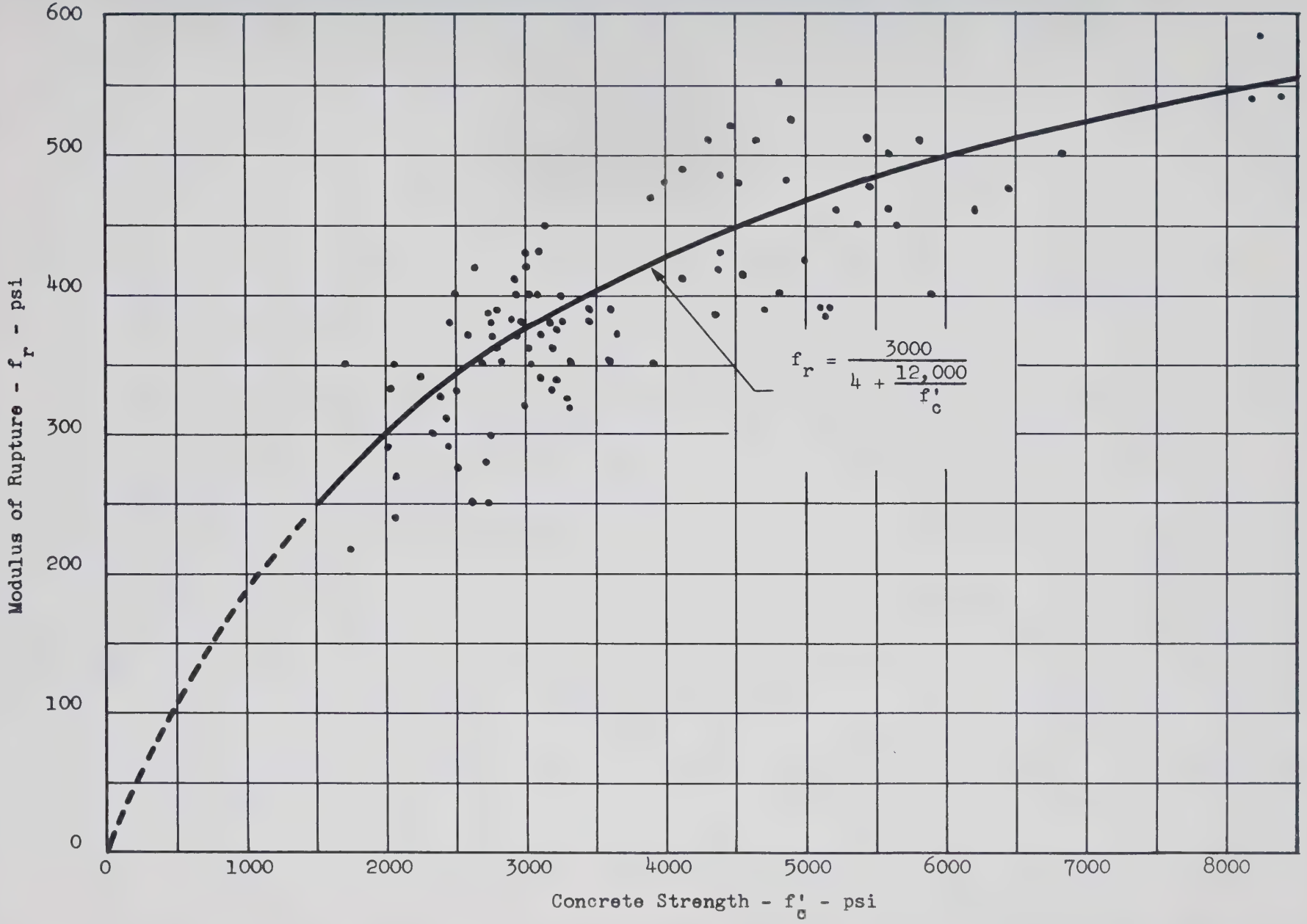

FIG. 2 COMPARISON OF MODULUS OF RUPTURE WITH CONCRETE STRENGTH MAXIMUM SIZE OF COARSE AGGREGATE: 3/8 in.

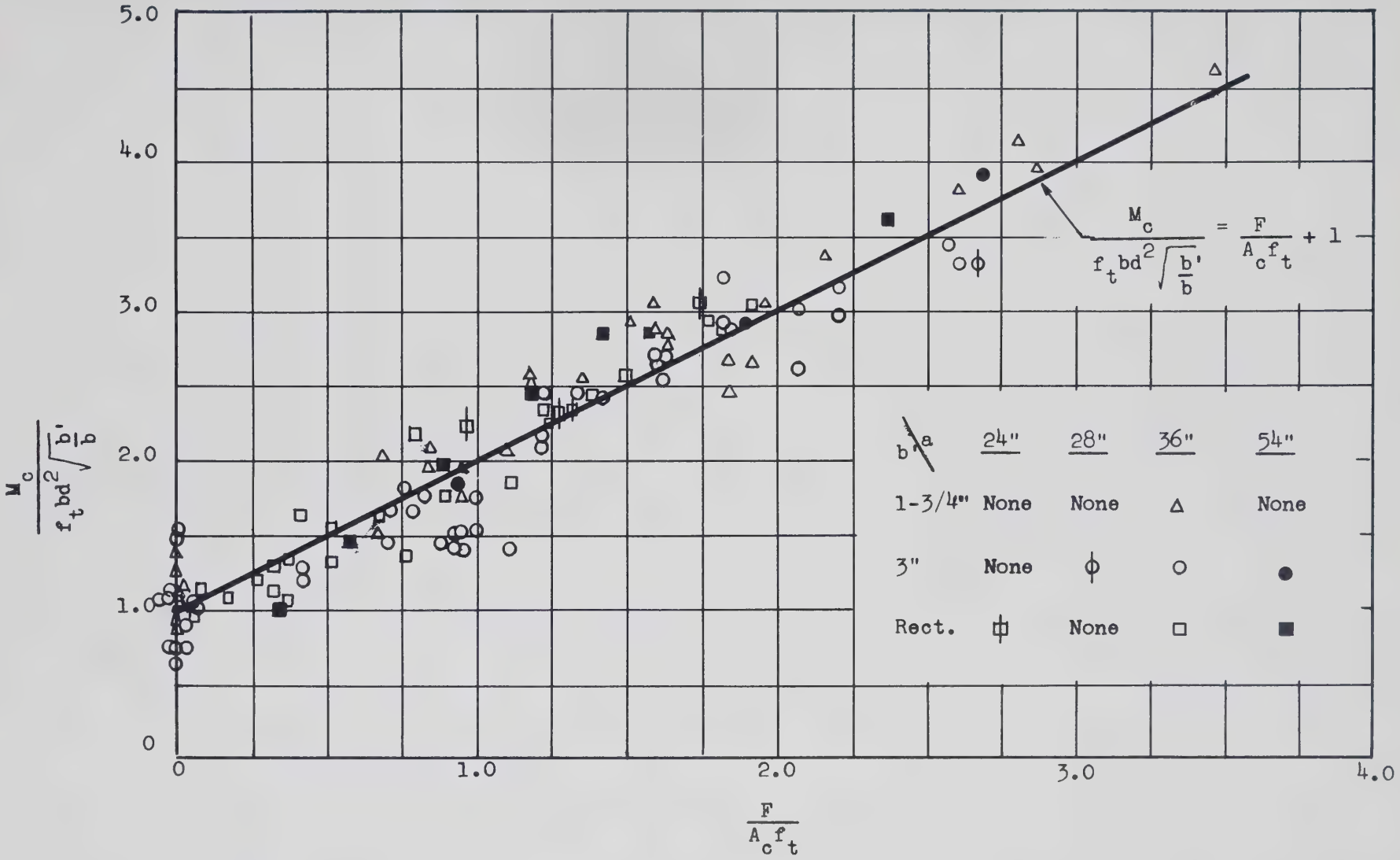

FIG. 3 COMPARISON OF MOMENT AT INCLINED TENSION CRACKING WITH AVERAGE COMPRESSIVE STRESS DUE TO PRESTRESS

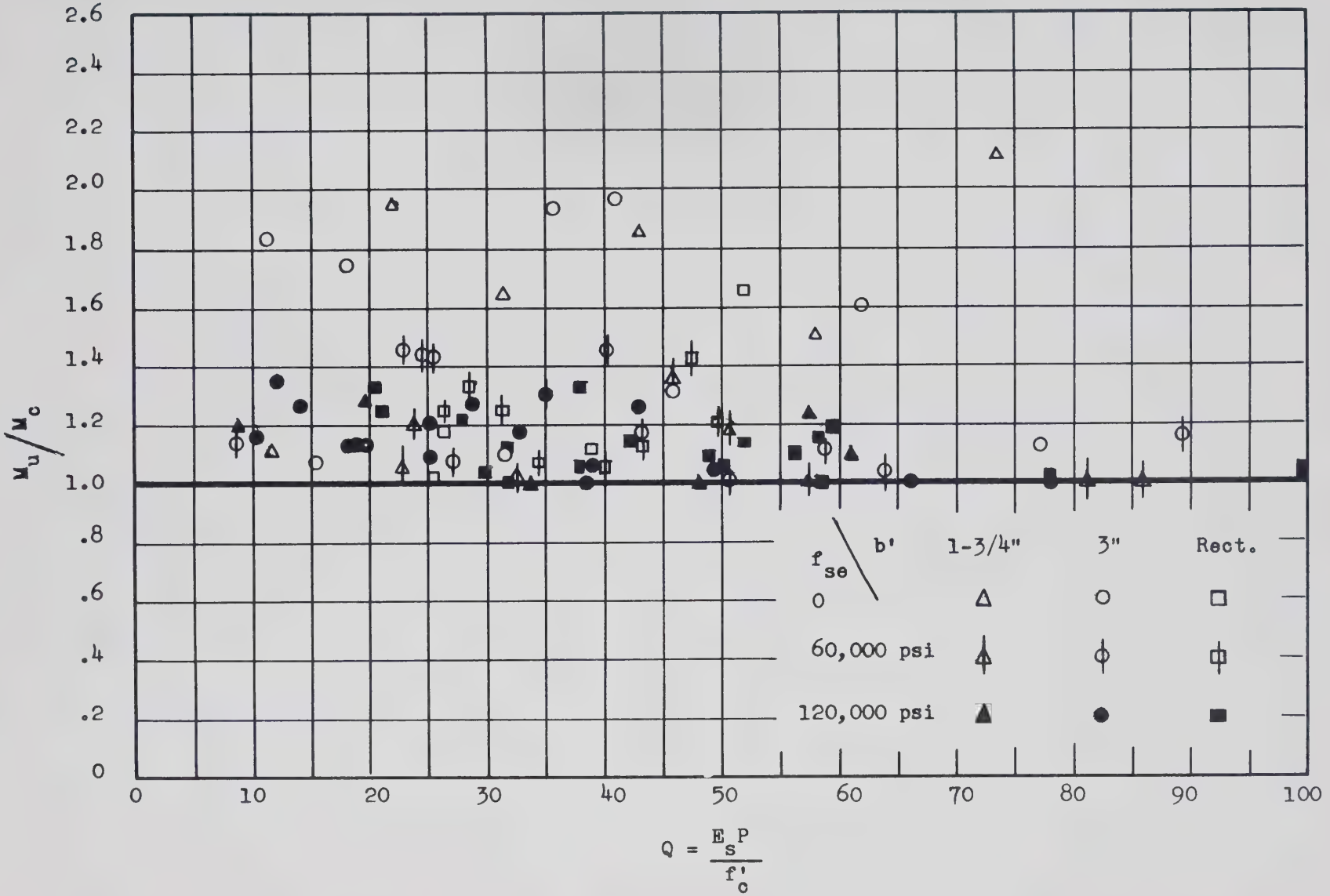

FIG. 4 COMPARISON OF ULTIMATE MOMENT WITH COMPUTED INCLINED TENSION CRACKING MOMENT

PART 3

PROGRESS REPORT

on

STUDIES OF TIME-DEPENDENT EFFECTS IN PRESTRESSED CONCRETE

by

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and

O. R. Gardi

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I. INTRODUCTION

1. Introduction

Following some pilot studies, two groups of tests relating to sustained loads on prestressed concrete beams were started, one in September 1954 and the other in January 1955, to study directly the combined effects of time-dependent variables. Each group comprised two beams and four cylinders. This report summarizes the data recorded in these tests and the studies which have been carried out as of August 1956.

Time-dependent deformations affect the behavior and possibly the strength of ordinary reinforced concrete beams mainly through the creep and shrinkage characteristics of the concrete. The steel is normally not stressed high enough to induce appreciable changes in stress and strain with time. In prestressed concrete, however, the problem is further complicated by the instability of the stress in the steel caused by elastic and time deformations and relaxation. Since our understanding of time-dependent effects in ordinary reinforced concrete is far from adequate, there is no clearly defined starting point for an investigation, theoretical or experimental, of the behavior of prestressed concrete beams under sustained loads. The only apparently infallible observation that carries over from such studies of reinforced concrete is that concrete creeps.

The differences in strain conditions and in the factors which govern the ultimate strength of ordinary and prestressed

concrete beams imply that time-dependent deformations are more significant and possibly critical in the latter type, especially in heavily reinforced or unbonded beams where the effective prestress plays an important role. Because of the relative certainty of strain and stress distribution in prestressed concrete, generally applicable theoretical analyses of time-dependent effects may be made even for fluctuating loads on the basis of assumptions describing the individual effects of the variables. However, the uncertainty of these assumptions makes it more desirable to attempt particular but simpler solutions of the problem. Consequently, the studies in this report are limited to numerical solutions of the time-dependent deflection problem on the basis of several simplifying assumptions. The nature of the problem and the attempted studies are described in the following sections.

2. Object and Scope

The primary purposes of this report are: (a) presentation of the data obtained from a series of four tests studying the influence of time-dependent variables on the behavior of prestressed concrete beams; and (b) comparison of the measured time-deflection curves with those based on various simplifying assumptions regarding the creep and relaxation characteristics of steel and concrete.

The data were obtained from long-time tests on two groups of specimens. Each group consisted of two beams and four companion cylinders. All beams were prestressed, post-tensioned, and

end-anchored. One beam of each group was loaded at its third points while the other was left unloaded. Two cylinders of each group were loaded to nominal stresses of 2000 and 1000 psi, and two were left unloaded.

The following table gives the ultimate strength of the concrete at the time of prestressing as indicated by tests on 6 by 12-in. cylinders for each group, the prestressing force, and the stresses imposed on the specimens. At this time both groups were over 60 days old. The applied stresses were intended to be about 40 per cent of ultimate concrete strength.

The time deflection data presented in this report extends to over 15,000 hours.

Specimen	Concrete Strength f'_c	Initial Prestress Force per Wire lbs	Total Initial Prestress Force lbs	Total Load lbs	*Top Fiber Stress (Comp.) psi	*Bottom Fiber Stress (Comp.) psi
G-1U	5500	3700	18500	0	0	2000
G-3U	5220	3700	18500	0	0	2000
G-1L	5500	3880	19400	3600	1800	0
G-3L	5220	3880	19400	3600	1800	0

* Reported stresses correspond to conditions immediately after loading.

3. Notation

The following notation is used in this report:

E_c = modulus of elasticity of concrete

E_s = modulus of elasticity of steel

f'_c = concrete strength as measured from 6 x 12 cylinders

t = time

T = tenure of test

ϵ_t = total concrete strain

ϵ_i = instantaneous concrete strain

Δ_i = instantaneous deflection

Δ_t = total deflection

α = ratio of shear span to total span

φ_i = instantaneous angle change

φ_t = total angle change

II. DESCRIPTION OF SPECIMENS AND MEASUREMENTS

4. Description of Specimens

(a) Beams. All tests were made on rectangular, post-tensioned, bonded beams with end anchorages. The two test groups, each of which comprised two beams and four companion cylinders were designated G-1 and G-3. The beams consisted of a rectangular concrete section nominally 4 by 6 in. in cross-section and 7 ft. in length (Figs. 1 and 2). The size of the grout channel was 2 by 2 in. Five high-tensile-strength unstraightened stress-relieved wire (Type OR-200)* having a nominal diameter of 0.195 in. were used in each beam.

The total prestress force on the loaded beam was 19400 lbs, while for the unloaded beam, it was 18500 lbs. The beams were reinforced in tension only. The reinforcement extended in a straight line from one end of the beam to the other. The center of gravity of the prestressing force was two inches above the bottom of the beam. One beam in each group was loaded with a total load of 3600 lbs applied at the third points by spring devices (Fig. 1). The letters "L" and "U" are used in the designation of the loaded and unloaded beams, respectively.

The reader is referred to the Fourth Progress Report* for details of manufacturing and loading of the test specimens.

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* Fourth Progress Report of the Investigation of Prestressed Concrete for Highway Bridges, Part 4, Engineering Experiment Station, University of Illinois.

(b) Cylinders. Four 4 by 16-in. companion cylinders were used to determine the effect of shrinkage and creep. Two cylinders in each group were loaded through springs in order to keep the load practically constant while the cylinders were allowed to creep and shrink (Fig. 3). One of these cylinders was loaded to the computed maximum beam stress of its group (2000 psi), while the other was loaded to half this stress (1000 psi). The other two cylinders of each group were left unloaded in an attempt to determine shrinkage strains.

5. Description of Measurements

(a) Strain Measurements

(i) Electrical Strain Gages. Electrical strain gages were used on Aluminum dynamometers to measure the tensioning force on the beams. The dynamometers were placed on the end of the wires opposite that at which the tension was applied. The tensioning force was determined by measuring the compressive strain in the dynamometers by means of two Type A-7 SR-4 electric strain gages.

Three equally spaced Type A-9 SR-4 electric strain gages were mounted on the loaded cylinders to measure longitudinal strains and to check the distribution of elastic strains during the process of loading in order to maintain a concentric load.

Two Type A-7 SR-4 electric strain gages were mounted on each of the three rods used in the cylinder loading frame. They were installed for strain measurements for the load-strain calibration of the rods.

Strains were read with a Baldwin Portable Strain Indicator.

(ii) Mechanical Strain Gages. The distribution of strain in the concrete over the depth of the beam as well as the creep and shrinkage strains in the cylinders were measured with a 10-in. Whittemore Strain Gage. The location of the gage lines are shown in Figs. 1 and 2. The values of strain for corresponding gage lines on the two sides of the beam were averaged to give one point on the strain distribution diagram.

(b) Deflection Measurements. Deflections were measured at the midspan of each beam with a 0.001 in. dial indicator. The location of the deflection dial at midspan can be seen in Figs. 1 and 2.

III. PRESENTATION OF TEST RESULTS

6. Beam Deflections

Plots of observed midspan deflections versus time for beams G-1 and G-3 are shown in Figs. 4 and 5, respectively. The readings extend to beyond 15,000 hours. The same time-deflection data are shown plotted against the logarithm of time in Figs. 18a and 18b.

On all these curves, the origin corresponds to the state of the beam before prestressing under dead load alone. The time intervals corresponding to the operations of prestressing and loading have been neglected on the plots since they cannot be conveniently shown to the time scales utilized.

The instantaneous deflections corresponding to prestress are shown by the ordinates of the curve at zero time. The instantaneous deflection of the loaded beams upon the application of load is indicated by the sudden drop in ordinate of the pertinent curves. The significant points in the life of the beam are marked on the curves.

7. Distribution of Strain over Depth of Beam

Strains which were measured across gage lines 1, 2, and 3 are presented in the form of strain distribution diagrams in Figs. 6 through 9. The location of the gage lines are shown in Figs. 1 and 2. The times at which the strain distributions were obtained are given at the top of each figure, while the magnitude of the strain is

shown at the bottom. All strain distribution diagrams are shown with reference to a plane corresponding to the state of the beam before prestressing, under dead load alone. Strain distributions recorded immediately after the applications of prestress and load are indicated on the figures. The strain distribution data does not extend to as long a time as the deflection data since some of the gage points were damaged and yielded unreliable readings after about 10,000 hours.

8. Cylinder Strains

The average of cylinder strains measured on the three gage lines shown in Fig. 3 are plotted against time in Figs. 10 and 12 for cylinders of groups G-1 and G-3, respectively. Compressive strains are shown as positive. The stress level to which each curve refers is indicated on the figures. The origins of all these curves refer to the condition of the cylinder before the start of the test at an age of about 60 days.

It was assumed that the shrinkage strains in the loaded cylinders was the same as the strain which the two unloaded cylinders underwent. Therefore, creep strains were obtained by eliminating the instantaneous and shrinkage strains from the total strains measured on the loaded cylinders. The creep strains so obtained are shown in Figs. 11 and 13 for groups G-1 and G-3, respectively.

9. Dimensionless Plots of Measured Concrete Strains

In Figs. 14 and 15 the abscissas indicate values of t/T expressed as a percentage, where t is the time of a specific reading

and T is 16,000 hours. The ordinates represent the creep strain at the time t expressed as a percentage of the total strain measured at 16,000 hours. They illustrate the "creep function" used in the computations based on cylinder strains which are discussed later in this report. Comparison of the curves obtained under two different applied stresses indicate that the assumption made with respect to the linearity of the relation between sustained stress and creep strain is permissible, especially in view of other scatter in the obtained data.

10. Humidity Measurements

The temperature and humidity were recorded with a Foxboro recorder. Wet and dry bulb temperatures were recorded at the test site and the humidity was determined by means of a conversion chart. Figure 16 is a plot showing the temperature and humidity fluctuations in the laboratory from 13 January 1955 to 12 September 1956.

IV. INTERPRETATION OF TEST RESULTS

11. Discussion of Data

(a) Cylinder Strains. Measured total cylinder strains are shown in Figs. 10 and 12. Desired creep strains are presented in Figs. 11 and 13. Comparison of creep strains at applied stresses of 2000 and 1000 psi are shown in Figs. 17 and 17b. In Fig. 17b, the strains are plotted versus the logarithm of time.

The cylinders were instrumented for mechanical readings of strain in order to obtain a creep function with the help of which the analysis of deflection of the beams could be carried out. Although the concrete mixes were virtually the same for both test groups, the curves of strain versus time obtained from each group at a stress level of 2000 psi were significantly different. Comparisons of the instantaneous strains, shrinkage strains, and creep strains under different stresses may be made directly by reference to Figs. 10 and 12. The instantaneous strains caused by the application of load are practically identical. The strains for the unloaded cylinders, which should reflect the effects of shrinkage, indicate that the unloaded cylinders of G-3 expanded, in other words they underwent "negative" shrinkage. On the other hand, observed shrinkage strains for G-1 implied a shortening of the cylinders as would be expected. Since there were two cylinders of each group, it is not justifiable to ascribe these conflicting results to experimental error. The uncontrolled humidity conditions in the laboratory might possibly have given rise to this discrepancy. Measurements on group G-1 cylinders were started in October 1954, and those of

group G-3 were started in February 1955. Unfortunately humidity records are available only for the period starting January 1955. Most of the difference between the two curves has taken place within the first three months. Humidity records for the period January 1955 to January 1956 show that this three month interval might have been somewhat drier for group G-1. However, the difference in humidity for the storage condition of the two specimens does not seem to be high enough to explain the disagreement in behavior. It should also be repeated here that the specimens were over 60 days old when the strain readings were started.

The curves of total strain versus time for G-1 and G-3 (Figs. 10 and 12) are also in disagreement. However, when it is considered that no effort was made to control all variables which affect the creep characteristics of concrete, it seems illogical to expect these curves to be identical.

Considering the loaded cylinders of group G-1, it can be seen from Fig. 14 that the rate of increase of creep strain is approximately the same for cylinders at the two stress levels. This applies also to loaded cylinders of group G-3 as shown in Fig. 15, although these rates are not alike for the two groups. For the cylinders loaded to 2000 psi, 80 per cent of the total strain increase has occurred within 30 per cent of the test duration for G-1, while the same proportion of the total strain increase took place at a time corresponding to 50 per cent of the test duration for G-3.

The ratios of the total measured deformations at 15,000 hours to the elastic deformations are given in the following table for both groups of cylinders:

Group	Stress psi	Total Deformation / Instantaneous Deformation
G-1	2000	3.2
	1000	2.9
G-3	2000	2.5
	1000	2.3

(b) Beam Strains

(1) Unloaded Beams. Distribution of measured strains for the unloaded beams G-1U and G-3U are shown in Figs. 6 and 7, respectively. As shown in the figures, the maximum instantaneous strain due to prestressing was 0.00047 for beam G-3U and was less than the strain of 0.00056 for beam G-1U. The maximum total strains after about 6000 hours are practically equal for both beams. This implies that the rate of increase of creep strain was higher for beam G-3U than G-1U. The ratios of maximum total strain after 9000 hours to the elastic strain are 2.5 and 2.9 for beams G-1U and G-3U, respectively.

(2) Loaded Beams. Distribution of measured strains for the loaded beams G-1L and G-3L are shown in Figs. 8 and 9. As indicated in the figures, the maximum instantaneous strain due to prestressing was less for beam G-3L than for G-1L. This is in agreement with the observed behavior of the unloaded beams. The effect of creep in the interval between prestressing and loading was higher for beam G-3L than for G-1L, which also agrees with the behavior of the unloaded beams.

The amount of instantaneous strain due to loading was greater for G-3L than for G-1L and the rate at which creep strains increased

after loading was higher for beam G-1L than that for beam G-3L. A possible explanation for these results is that the eccentricity of the prestressing force was greater in beam G-3L than in beam G-1L, with the result that the moment due to the prestressing force was greater in G-3L. This may also explain why the rate of increase of creep strain after prestressing and before loading was higher for beam G-3L. Naturally, this relation is reversed after loading.

The measured changes in strain one-half below the top and one-half inch above the bottom of the beams are given in the following table, with compressive strains listed as positive.

Beam	Location	Strain Changes in inches/inch x 10^6				
		Prestress Only (Instantaneous)	After Prestressing*	Load Only (Instantaneous)	After 9000 hrs	Total at 9000 hrs
G-1L	$\frac{1}{2}$" from Top	+155	- 20	+320	+845	+1330
	$\frac{1}{2}$" from Bot.	+560	- 20	-355	+300	+ 505
G-3L	$\frac{1}{2}$" from Top	+ 55	- 70	+390	+660	+1025
	$\frac{1}{2}$" from Bot.	+495	-290	-400	+190	+ 555

* Due to time-dependent effects only.

The ratios of the maximum total strain due to load and time-dependent effects to the maximum instantaneous strain due to load are on the order of 3.7 and 2.7 for beam G-1L and G-3L, respectively.

(c) Beam Deflections

(1) Unloaded Beams. Midspan deflections for the unloaded beams are compared in Fig. 18a. The elastic deflections due to prestressing are approximately the same for both beams, but the rate of increase of deflection due to time effects after prestressing is higher for beam G-3U than for G-1U. This is consistent with measured distributions

of strain in the beams as discussed in the preceding paragraph. The ratios of total deflection at 15,000 hours to the instantaneous deflection are 2.2 and 2.9 for G-1U and G-3U, respectively.

(2) Loaded Beams. Midspan deflections for the loaded beams are compared in Fig. 18b. The instantaneous deflections due to prestressing are the same for both beams. The rate of increase due to creep after prestressing and the amount of instantaneous deflection due to loading only are greater for beam G-3L. The rate of increase of deflection due to time effects after loading is higher for G-1L. These conditions are consistent with the distribution of strains in the beams as mentioned earlier in this section.

The recorded midspan deflections for the loaded beams are given in the following table for the purpose of comparing the deflections taking place during various stages in the course of the tests. Upward deflections are listed as positive. The total deflection at 15,000 hours is measured from a datum plane corresponding to the position of the beam before application of prestress.

Beam	Changes in Midspan Deflection in inches				
	Due to Prestress	Interval (1)*	Due to Load	Interval (2)**	Total at 15,000 hrs
G-1L	+0.055	+0.002	-0.078	-0.063	-0.085
G-3L	+0.056	+0.023	-0.094	-0.052	-0.067

* Interval (1): Period between prestressing and loading.

** Interval (2): Period between loading and 15,000 hours.

It is to be noted that the greater part of the difference in the deflection of the two beams appears to have taken place in the interval between prestressing and loading. This interval was 138 hours for beam G-1L and 120 hours for beam G-3L.

12. Comparison of Behavior of Loaded and Unloaded Beams

The deflection-time curves presented in Figs. 4 and 5 indicate that the ratio of total to instantaneous deflection is greater for the unloaded than for the loaded beams. The instantaneous deflection is regarded as that due to the prestress for the unloaded beams and as that due to the applied load for the loaded beams. This phenomenon is an important characteristic of the behavior of prestressed concrete beams and can be explained by reference to the following idealized cases.

Consider first the effects of concrete creep in an ordinary reinforced concrete beam. Line DB on Fig.19a describes an arbitrary strain distribution corresponding to conditions immediately after the application of a design load on such a beam. The instantaneous angle change is $\varphi_i = (\epsilon_i + \epsilon_s)/d$. If it is assumed that after a certain length of time the ratio of total to instantaneous concrete strain is four, then point B moves to B' as shown in the figure. Ignoring the relatively small change in the steel strain resulting from the reduced internal moment arm and increase in internal compressive force, the resulting strain distribution can be approximated by the line DB'. The associated total angle change, φ_t, is then given by the ratio $(\epsilon_s + 4\epsilon_i)/d$. Since ϵ_s is usually of the same order of magnitude as ϵ_i, the increase in angle change is about 2.5 times the instantaneous deflection, as compared to a fourfold increase in the concrete strain. Thus, creep does not influence the deflection of ordinary reinforced concrete beams directly because the time-rotation of the strain

distribution takes place about a pivot point approximately at the level of the reinforcement while the rotation due to loading occurs about a point on the neutral axis at the time of loading.

Consider next the condition in a prestressed concrete beam subject only to the stresses due to the prestress itself; that is, the dead load stresses are negligible as they were in the unloaded beams reported herein. Line EG on Fig. 19b represents an arbitrary strain distribution over the depth of such a beam immediately after prestressing. If the ratio of total to instantaneous strain after a certain length of time is again assumed to be four, and if the losses in prestress are ignored, then EG' represents the strain distribution after this length of time. The instantaneous and total angle changes are given by the ratios ϵ_i/h and $4\epsilon_i/h$, respectively, and the increase in angle change is thus directly proportional to the increase in concrete strain.

If a load is applied to the beam prestressed as shown in Fig. 19b so that the strain distribution becomes as shown on Fig. 19c, the instantaneous angle change due to the application of the load is $2\epsilon_i/h$. The adoption of a factor of four on total strain as compared to instantaneous strain results in a total angle change given by the ratio $4\epsilon_i/h$. That is, the increase in angle change is only half as much as the increase in concrete strain. Similarly, for a strain distribution after loading as shown in Fig. 19d, the ratio of total to instantaneous angle change would theoretically be unity whatever the

change in concrete strain since the uniform strain distribution would cause no further angle change with time.

For still another case, such as that shown in Fig. 19e where the strain distribution after loading is indicated by the line IH, the ratio of total to instantaneous angle change would be less than unity since larger creep strains at the bottom fiber would result in an upward deflection. As indicated by these special cases, the increase in angle change due to creep in loaded beams is always less than the increase in concrete strain due to creep.

The relation between long-time and instantaneous deflection would be similar to that for angle change except for the fact that the angle change may vary significantly along the span length. For example, line ABCD in Fig. 20a designates the angle change distribution for a simply-supported weightless beam with straight tendons. It is assumed that the strain distribution in this beam due to prestress is as shown in Fig. 19b, and that the beam is loaded so as to have a strain distribution like that shown in Fig. 19c at sections of maximum moment. It follows that the strain distribution at the inflection point corresponds to Fig. 19d. The strain distribution is similar to that shown in Fig. 19e at sections between the inflection point and the support, and it is represented by Fig. 19b at the support. After a certain length of time the angle change distribution may be assumed to be represented by the line A'B'C'D' in Fig. 20a which is consistent with the preceding discussion of angle changes due to creep if prestress losses are neglected. According to this idealized diagram, the

time-curvature is concave upward in the region between the two inflection points, while it is concave downward in the outer portions of the beam. Thus, the downward deflection of the beam at midspan is reduced, and this reduction increases with the ratio of the shear span to the total span in beams where the loads are arranged as shown in Fig. 20a. The relation between the ratios of total to instantaneous deflection and strain for different values of α for the case shown in Fig. 20a is shown in Fig. 21.

Curves ABC and A'B'C' on Fig. 20b designate the instantaneous and total distributions of angle change in a uniformly-loaded simply-supported beam with straight tendons. It is again assumed that strain conditions at midspan are represented by Fig. 19c, while the strain conditions over the reactions are represented by Fig. 19b. Evidently, the relation between long-time deflection to strains for this case is also modified in a manner similar to that described for Fig. 20a.

Thus, the ratio of total to instantaneous deflection for the loaded beams G-1L and G-3L in which the strain and angle change distributions are approximately as shown in Fig. 20a is less than the ratio for the unloaded beams in which the angle change is uniformly distributed along the span and the strain distribution corresponds to that shown on Fig. 19b.

13. Description of Analysis

The following general assumptions were made:

(a) Creep strains are linearly proportional to the sustained stresses.

(b) Shrinkage strains are independent of stress and uniform over the depth of the beam.

(c) The rate of strain at any time under any intensity of stress is independent of the previous stress and strain history.

No attempt was made to use a general creep function for the concrete. The relationships between concrete strain and time was based on observations made in connection with the tests. Three such relationships were tried in computing time-deflections:

(1) Creep function determined from companion cylinders of the group for which the computations are carried out. (Figs. 11 and 13).

(2) Creep function determined from companion cylinders of group G-1 only. (Fig. 11).

(3) Creep function derived from measured strains on beam G-3U. (Fig. 22).

The magnitude of the shrinkage strains, where needed, were obtained from data pertaining to the beam under consideration.

The relaxation loss in the steel was computed from results of relaxation tests carried out on similar wires. Since the prestressing wires in the beams had different initial stresses than any of the wires in the relaxation tests, the relaxation of the wires in the beams was obtained by linear interpolation between the relaxation of a wire with a slightly higher initial stress and a wire with a slightly lower initial stress. The relaxation loss thus obtained was very small

since the initial stress level was on the order of 50 per cent of the ultimate strength of the wire. Although this loss was included, it had practically no effect on the deflection computations.

The modulus of elasticity of concrete was assumed to be independent of time and to be given by the following expression:

$$E_c = \frac{30,000,000}{5 + \frac{10,000}{f'_c}}$$

Changes in beam strain and related losses in prestress with time were considered in steps: (1) the strain distribution at a certain time was determined on the basis of an assumed creep function; (2) the corresponding loss in steel stress during the same interval was based on the creep and shrinkage strains at the level of the steel relaxation; (3) the strain distribution was corrected for the "elastic" change in strain caused by the loss in prestress. Strain changes in the following selected intervals of time were computed similarly. Thus, it was possible to construct the angle-change diagram and compute the corresponding midspan deflection. Repetitions of such calculations resulted in a plot of deflection versus time which was compared with test data.

The loss in prestress resulting from creep in the concrete after a time of about 15,000 hours was on the order of 20 per cent for the unloaded beams and 15 per cent for the loaded beams. However,

since the creep strains were large compared with the "elastic" strains due to loss in prestress, the effect of this loss on the computed deflection was small.

14. Comparison of Measured and Computed Deflections

(a) Computed Deflections Based on Creep Function (1).

Comparisons of measured midspan deflections and those computed using creep function (1) based on data from the respective companion cylinders are presented graphically in Figs. 23, 24, 25, and 26. The values in the first line of the following table refer to the ratios of total to instantaneous strain as measured on the pertinent cylinders stressed to 2000 psi. The values in the second and third lines are measured and computed ratios of total to instantaneous midspan deflection, respectively. The instantaneous deflection is taken as the upward movement at midspan due to prestress for the unloaded beams, and as the downward movement at midspan due to application of load for the loaded beams. In both cases, the total deflection is the sum of the time- and instantaneous deflections at midspan. For the loaded beams, it does not include the instantaneous upward deflection due to prestress.

	G-1U	G-3U	G-1L	G-3L
1. Measured ϵ_t/ϵ_i	3.2	2.5	3.2	2.5
2. Measured Δ_t/Δ_i	2.2	2.9	1.8	1.6
3. Measured/Computed Δ_t	0.76	1.20	0.83	0.92

A comparison of the ratios given on the first and second lines of this table indicates that increase in deflection of the unloaded beams is of the same order as the increase in concrete strain due to creep, while the increase in deflection of the loaded beams is definitely less than the increase in concrete strain. The reasons for this phenomenon were explained in Section 12 of this report. If the observed creep function and modulus of elasticity for concrete represented precisely the behavior of the concrete in the beam, the ratios in the first and second lines would be identical for the unloaded beams except for some reduction in the measured ratios caused by the loss in prestress.

The agreement between computed and measured deflections is relatively poor as is indicated also in Figs. 23, 24, 25, and 26. Except for beam G-3U the computed deflections are higher than the measured values.

(b) Computed Deflections Based on Creep Function (2). Since the creep strain readings from the cylinders of group G-1 appeared to be more consistent, and since both beam groups had been cast from like concrete mixes, the computations for beams G-3 were repeated using the creep function derived from cylinders of group G-1 only. Deflections thus computed are compared with test data on Figs. 27 and 28. At 15,000 hours, the ratios of measured to computed midspan deflection are 1.09 and 0.88 for beams G-3U and G-3L, respectively. This

represents little improvement over the computations based on the creep function derived from cylinders of group G-3. However, the shape of the deflection-time curve is defined more successfully for beam G-3U (Compare Figs. 25 and 22).

(c) Computed Deflections Based on Creep Function (3). Another attempt was made at determining a satisfactory creep function by deriving it from strain measurements on the beam. The data obtained from the bottom gage lines of beam G-3U were utilized for this purpose. Creep strains were computed from the strain readings by subtracting the shrinkage strains as indicated by the unloaded cylinders, adding the "elastic" strains corresponding to the loss in prestress for that interval of time, and modifying the resulting strain to give the creep under 2000 psi on the basis of the assumption regarding the linearity of the relation between creep and applied stress.

Figures 29, 30, 31, and 32 compare the measured data with the deflections based on this creep function. Although the computed deflections are fairly high compared to the measured values, except for beam G-3U, the shapes of the measured deflection-time curves are similar to the computed curves in every case.

15. General Discussion of Test Results

Two critical questions can be asked regarding the combined effects of time-dependent variables on prestressed concrete beams:

(1) Do the effects of time-dependent variables become negligible after a certain length of time? (2) Can these effects be predicted?

The studies and tests presented in this report are far from being adequate, and neither question can be answered with certainty. However, certain trends observed in the studies deserve mention despite the limited number of tests.

The four beams have been under test for a period of about two years. Curves of measured midspan deflections plotted versus the logarithm of time (Figs. 18a and 18b) or versus time (Figs. 4 and 5) have not "leveled off" within this period. The deflections were still increasing at the end of 15,000 hours, although at a relatively slow rate.

There has been observed considerable difference between the deflection-time curves of beams having ostensibly identical properties.

The time-deflection of the beams depended mainly on the creep characteristics of the concrete. The effects of losses in prestress were relatively low. Because of a compensating distribution of angle-changes due to creep strains, the time-deflections of the loaded beams were less than those of the unloaded beams.

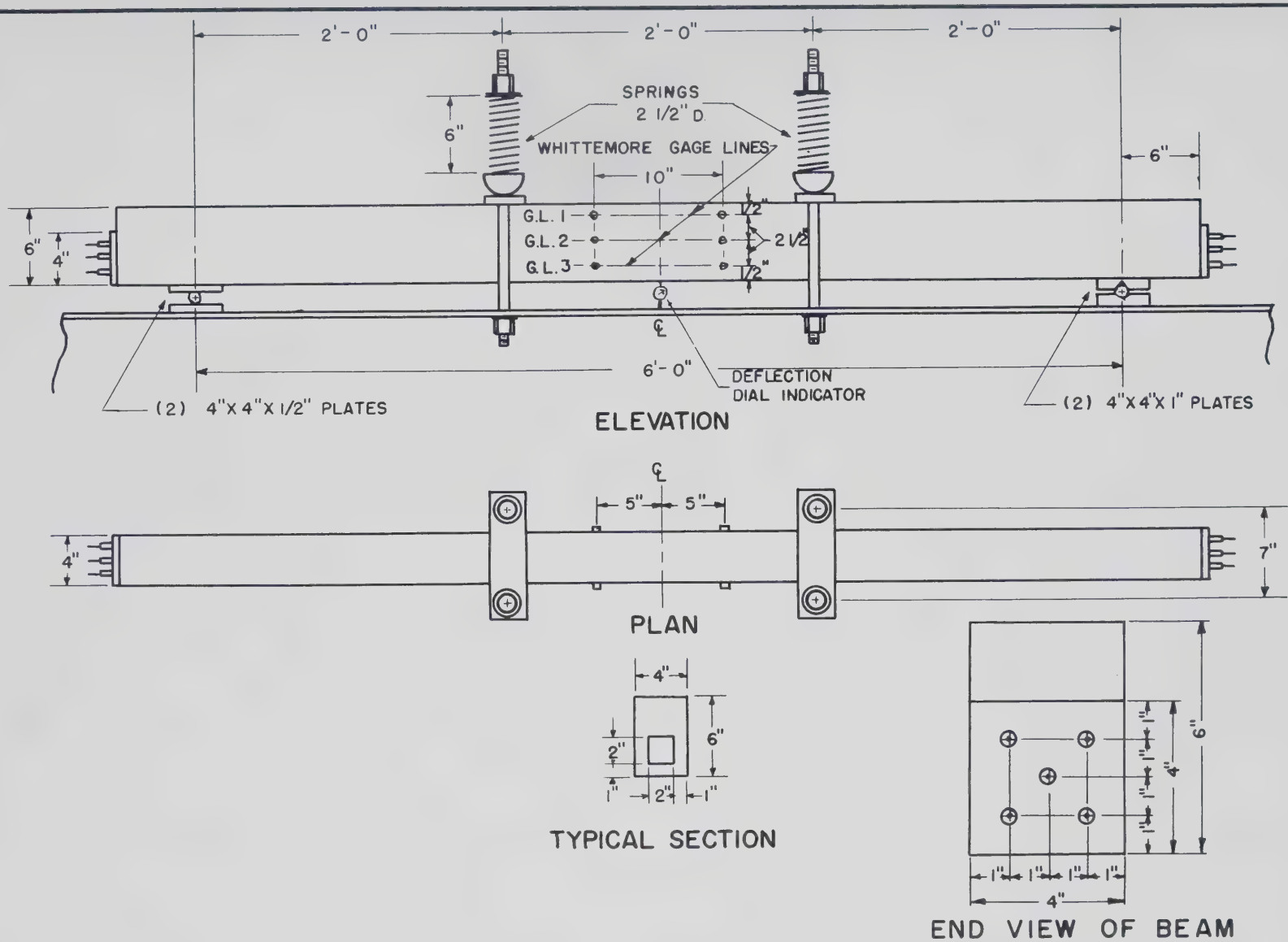


FIG. 1 DIMENSIONS AND LOADING ARRANGEMENT FOR BEAMS G-1L AND G-3L

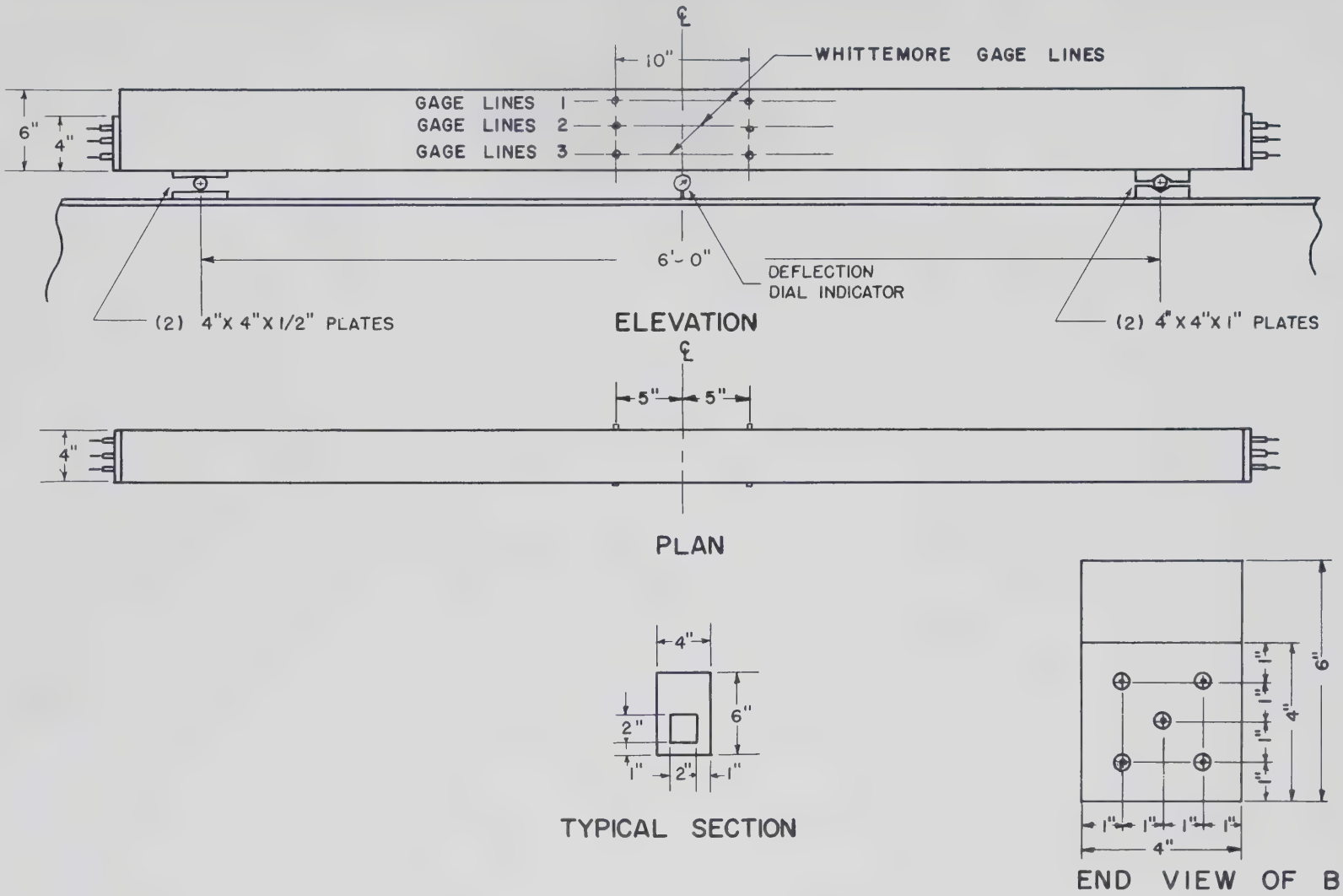


FIG. 2 DIMENSIONS FOR BEAMS G-1U AND G-3U

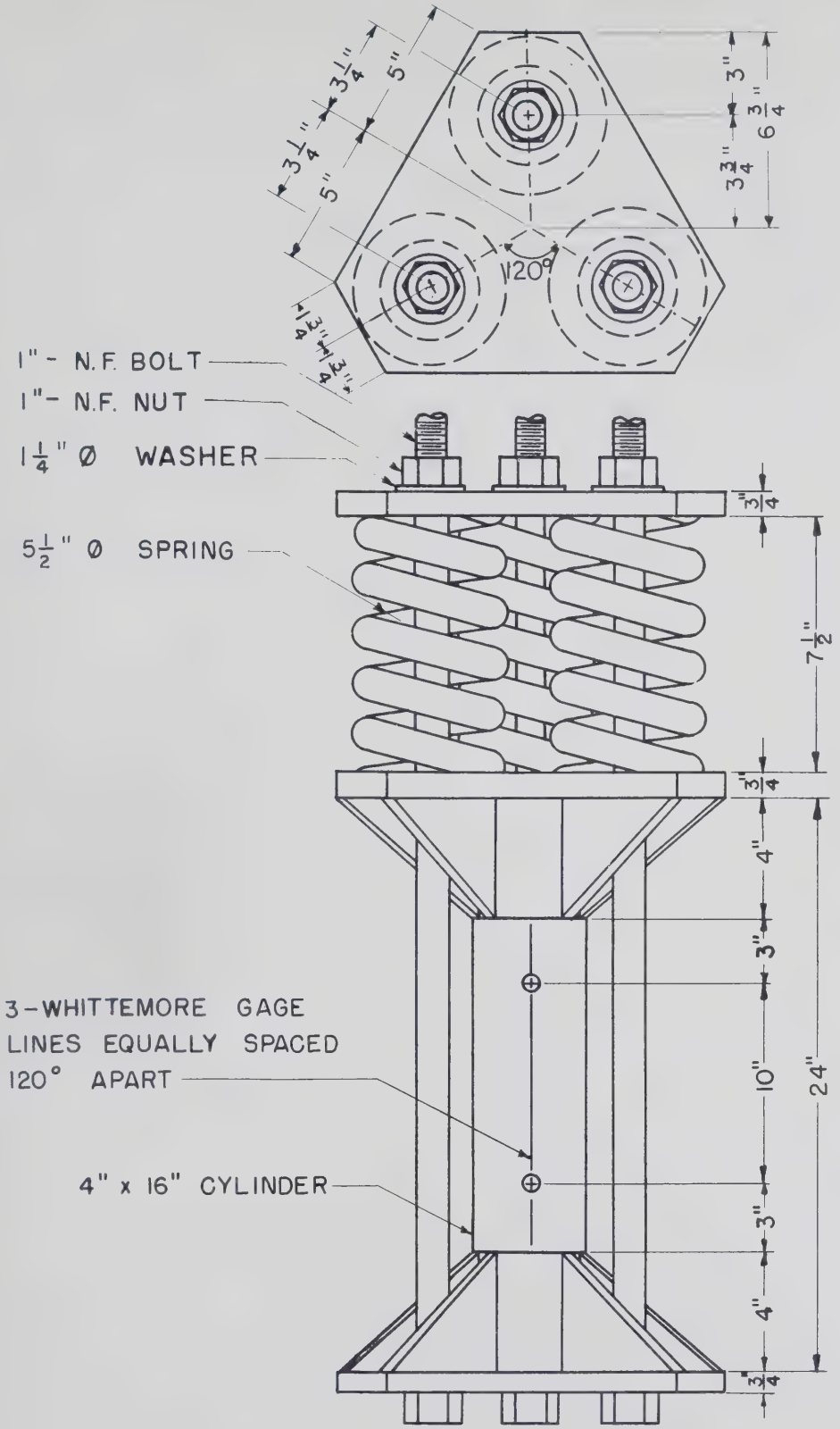


FIG. 3 DIMENSIONS AND LOADING ARRANGEMENT FOR THE 4"x 16" CYLINDERS

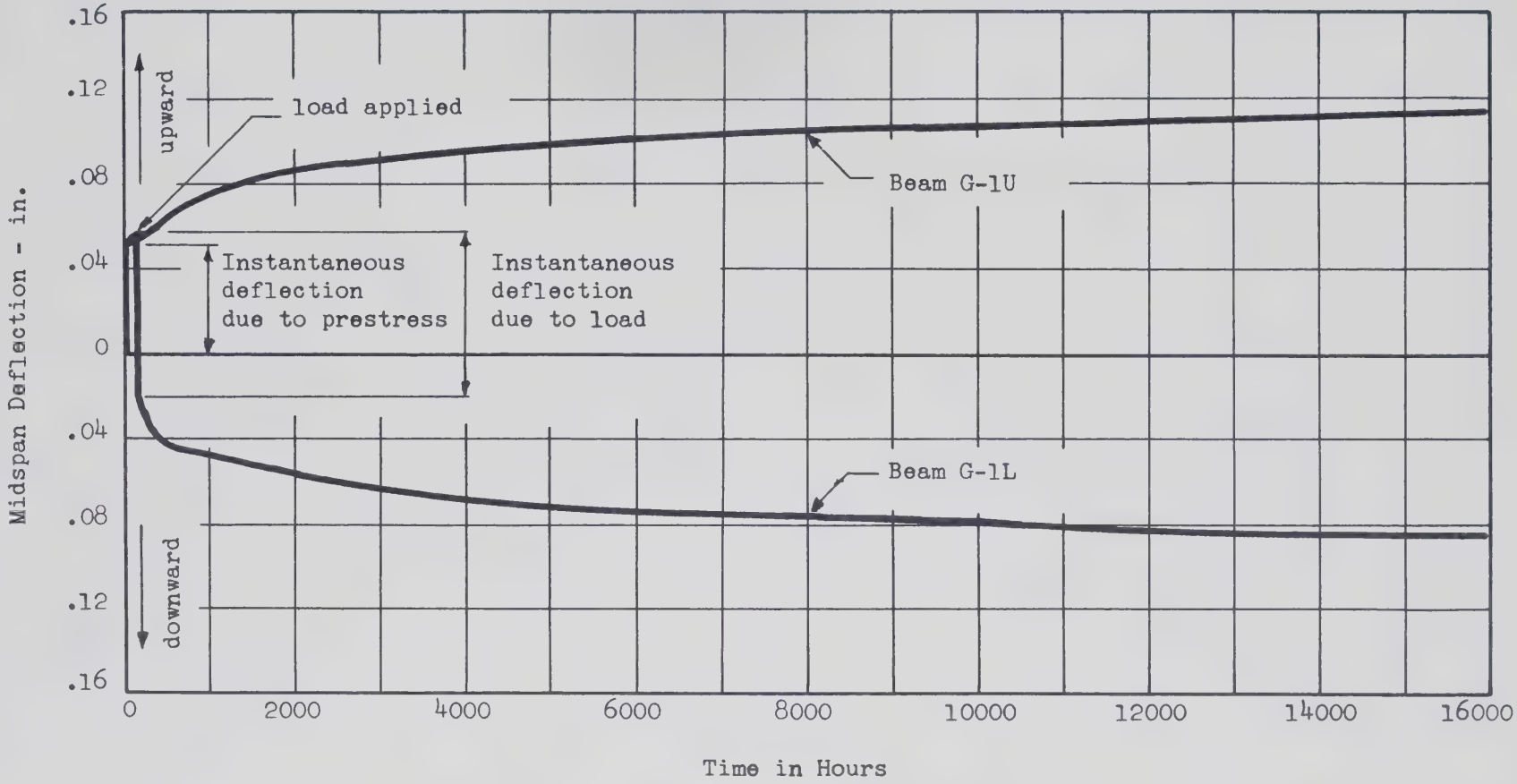


FIG. 4 MIDSPAN DEFLECTION VERSUS TIME FOR GROUP G-1

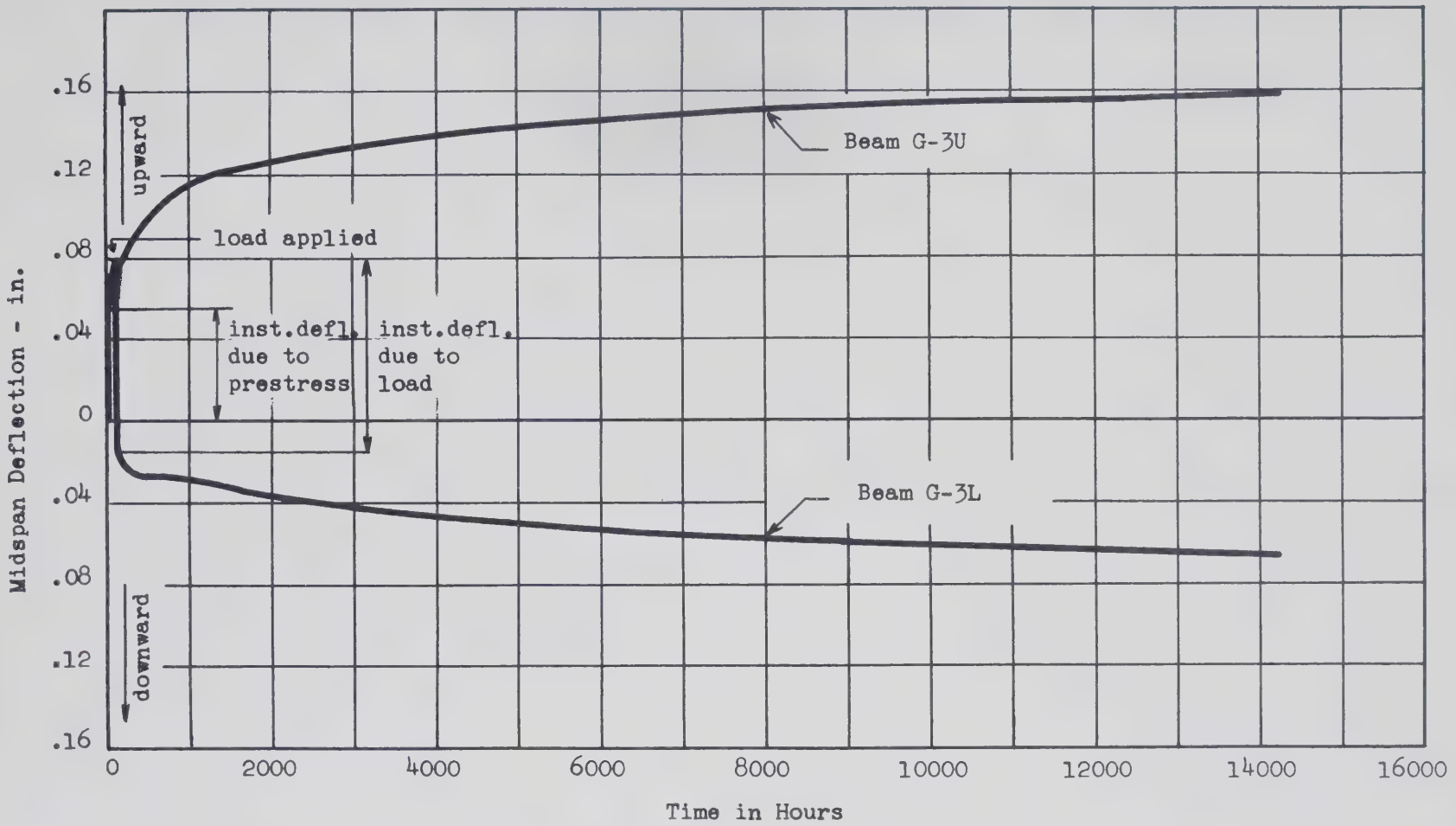


FIG. 5 MIDSPAN DEFLECTION VERSUS TIME FOR GROUP G-3

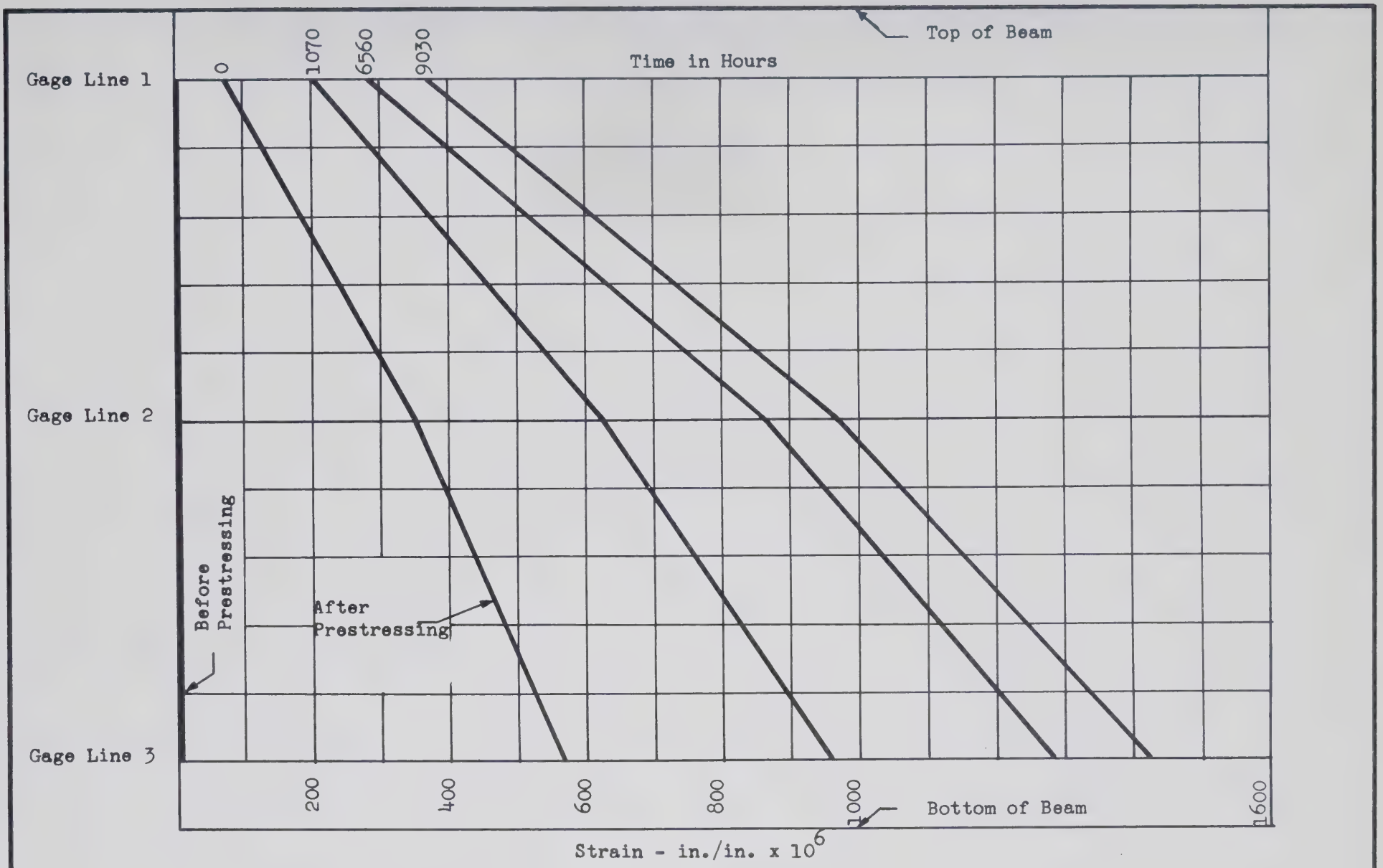


FIG. 6 DISTRIBUTION OF STRAIN OVER DEPTH OF BEAM G-1U

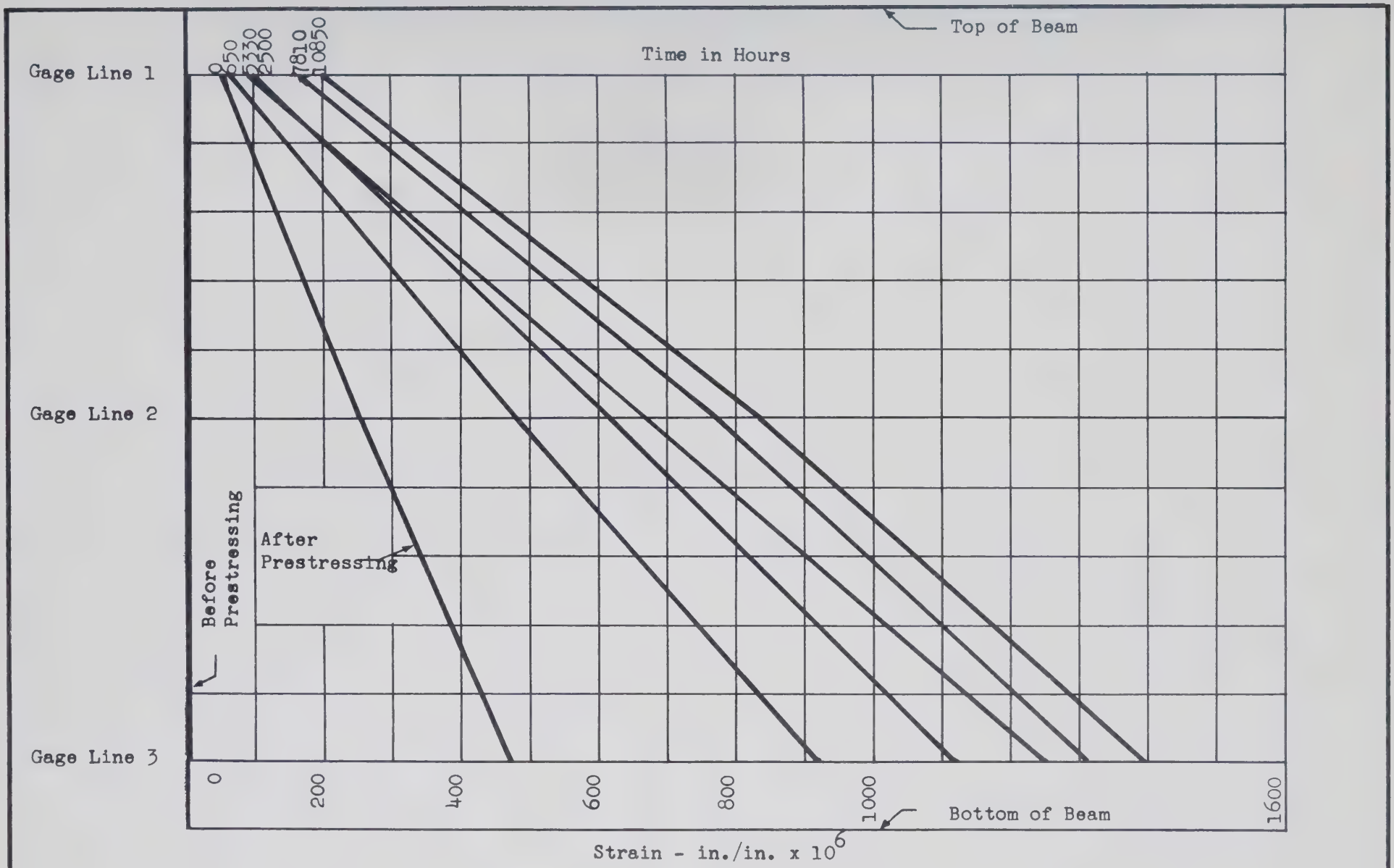


FIG. 7 DISTRIBUTION OF STRAIN OVER DEPTH OF BEAM G-3U

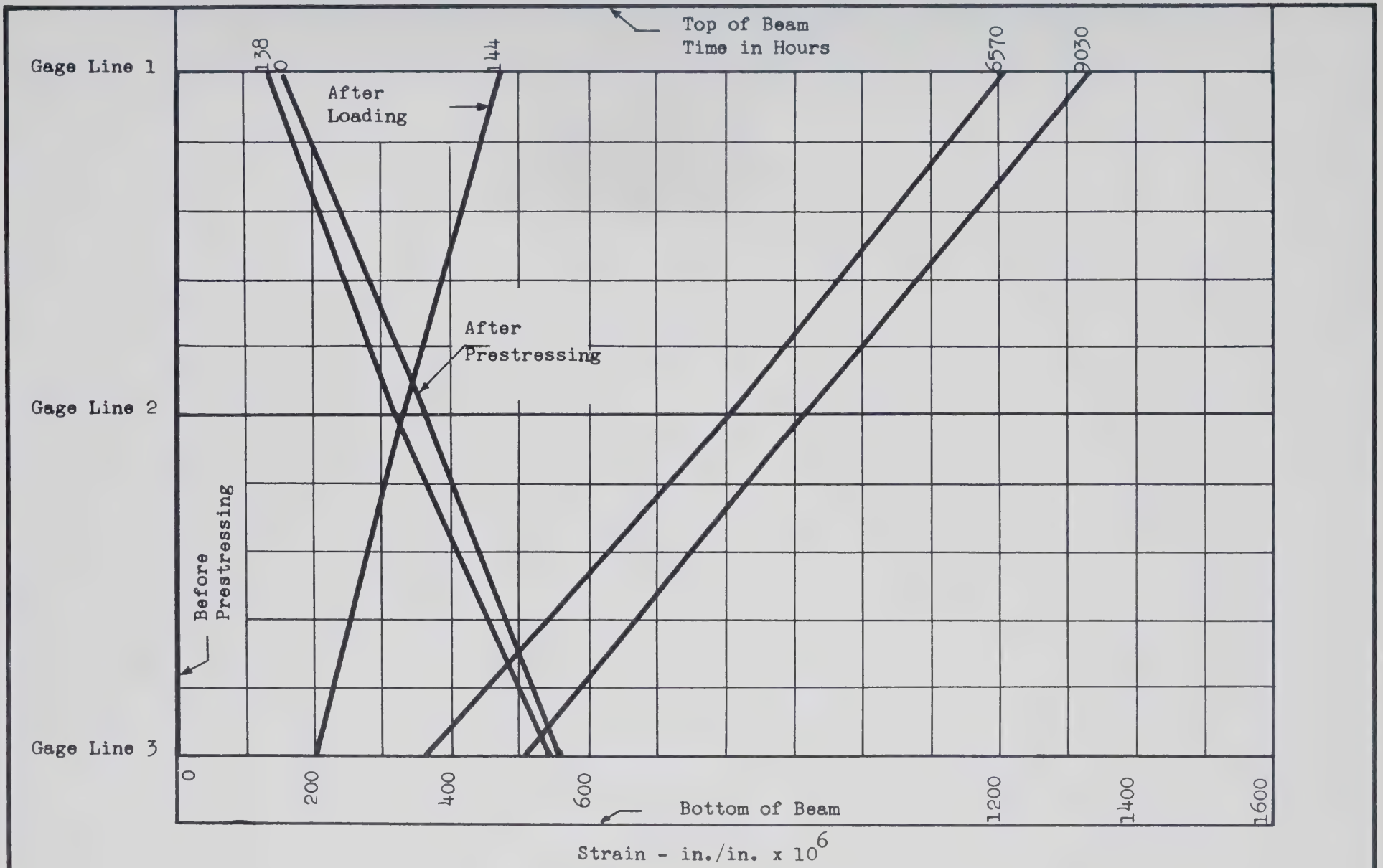


FIG. 8 DISTRIBUTION OF STRAIN OVER DEPTH OF BEAM G-1L

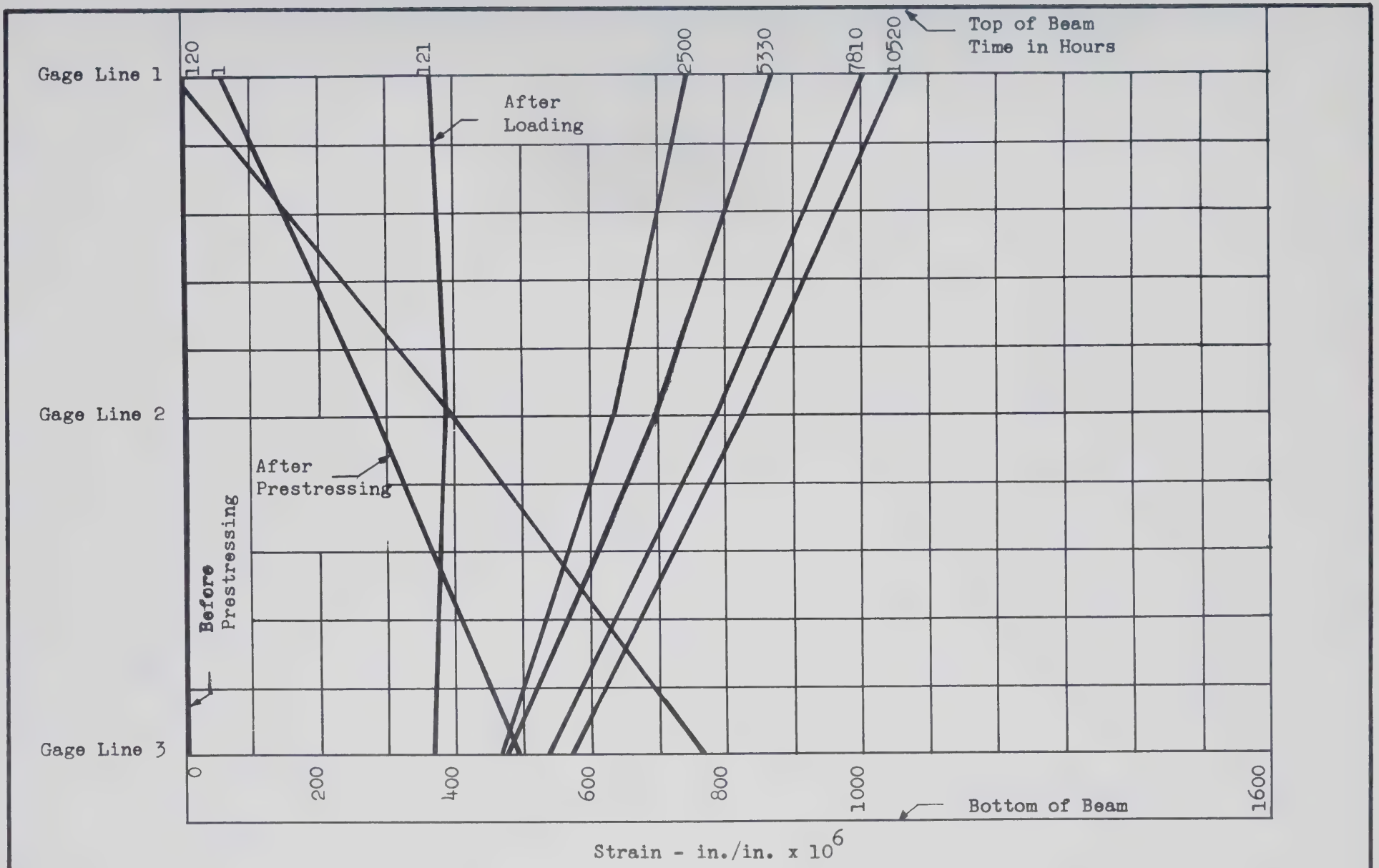


FIG. 9 DISTRIBUTION OF STRAIN OVER DEPTH OF BEAM G-3L

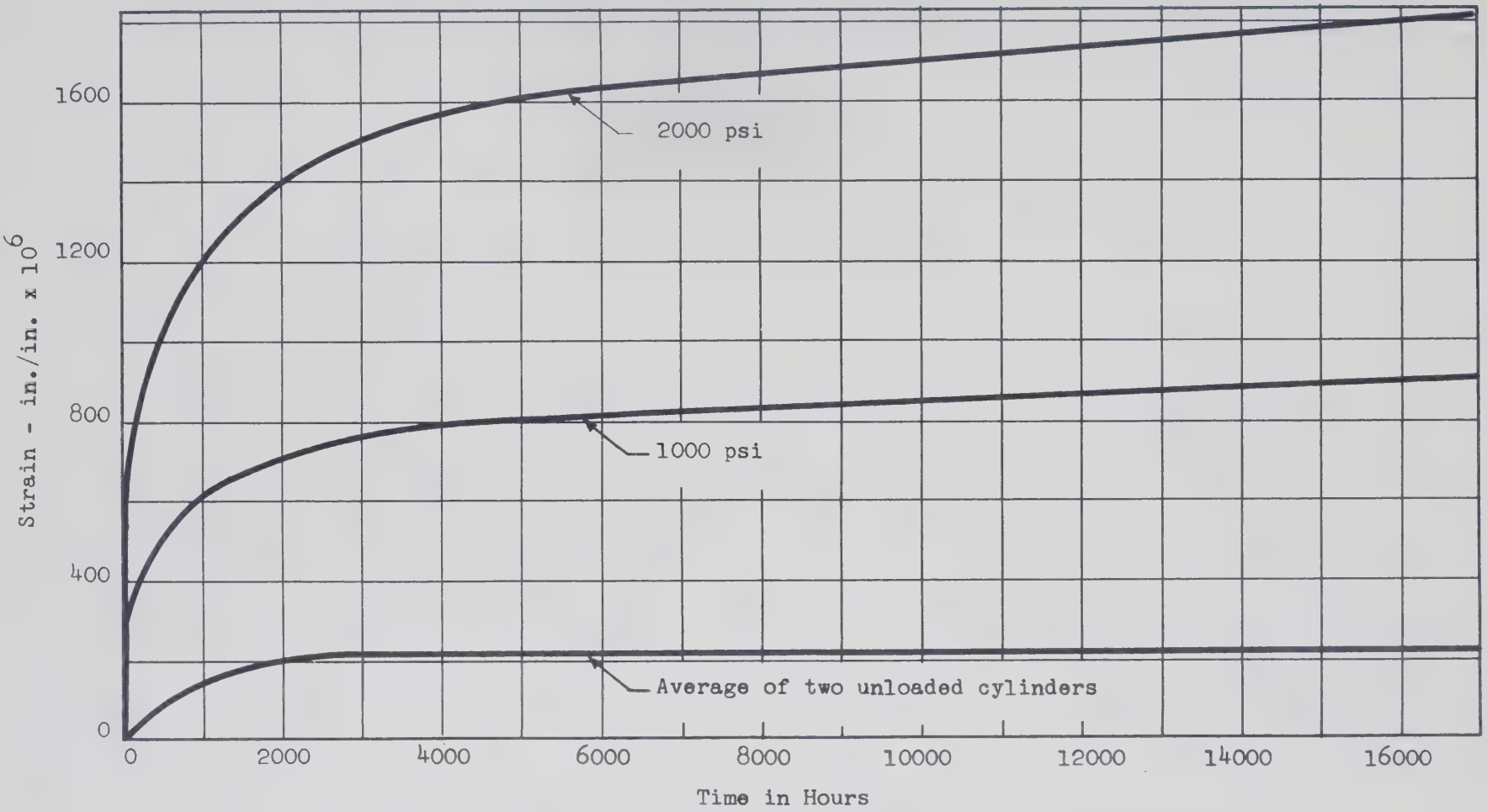


FIG. 10 TOTAL MEASURED STRAIN VERSUS TIME FOR 4 BY 6 IN. CYLINDERS OF GROUP G-1

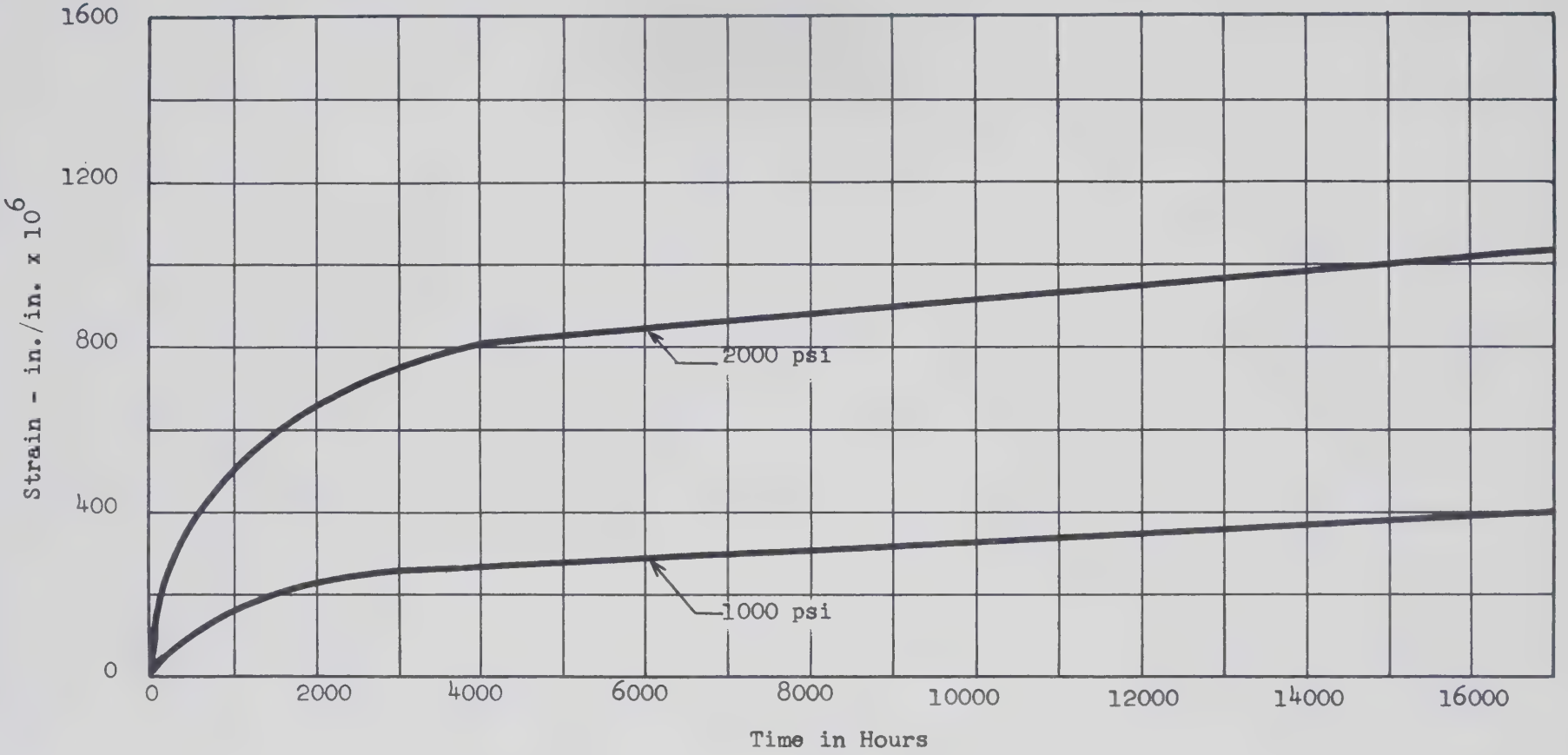


FIG. 11 CREEP STRAIN VERSUS TIME FOR 4 BY 16 IN. CYLINDERS OF GROUP G-1

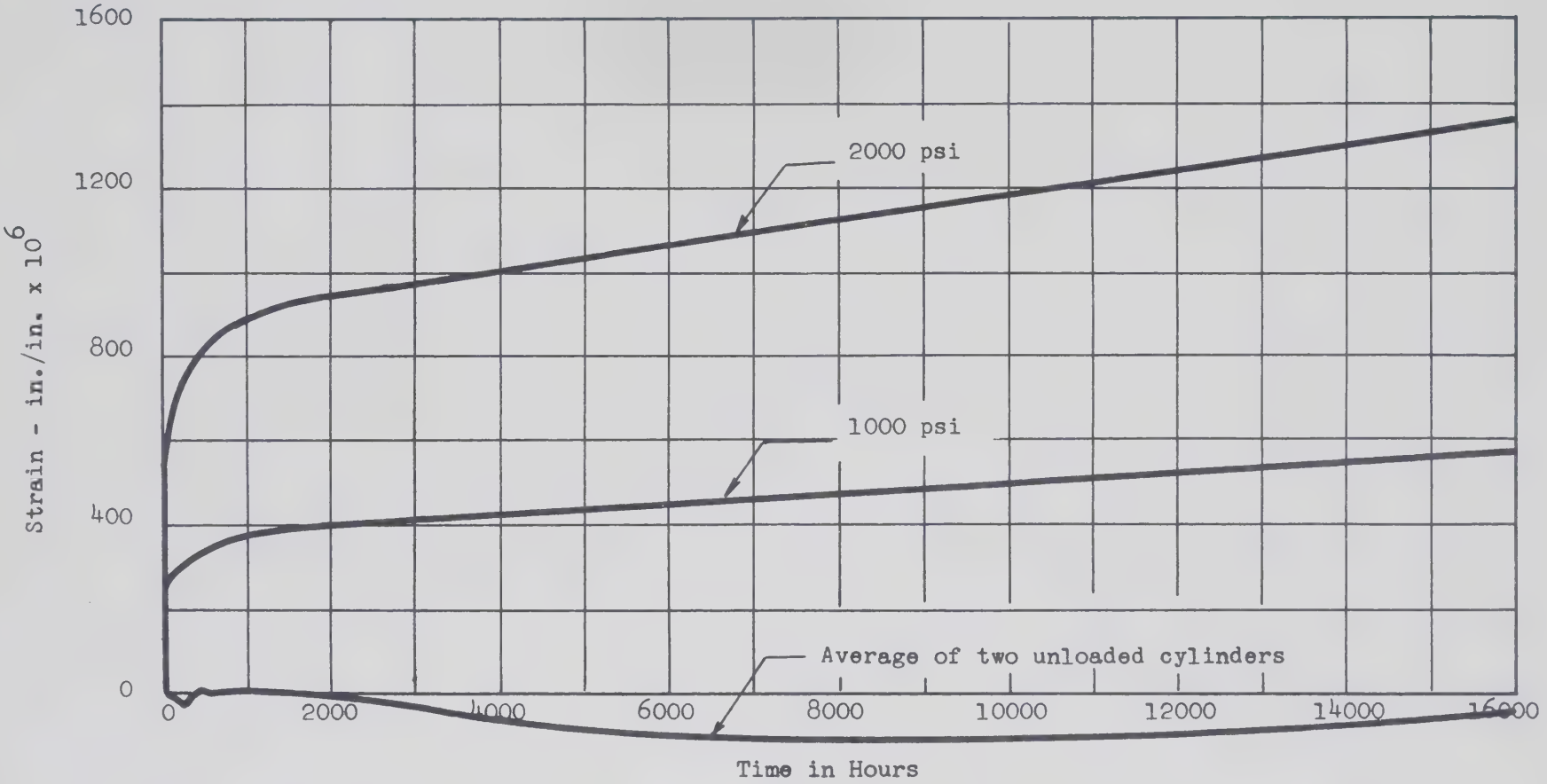


FIG. 12 TOTAL MEASURED STRAINS VERSUS TIME FOR 4 BY 16 IN. CYLINDERS OF GROUP G-3

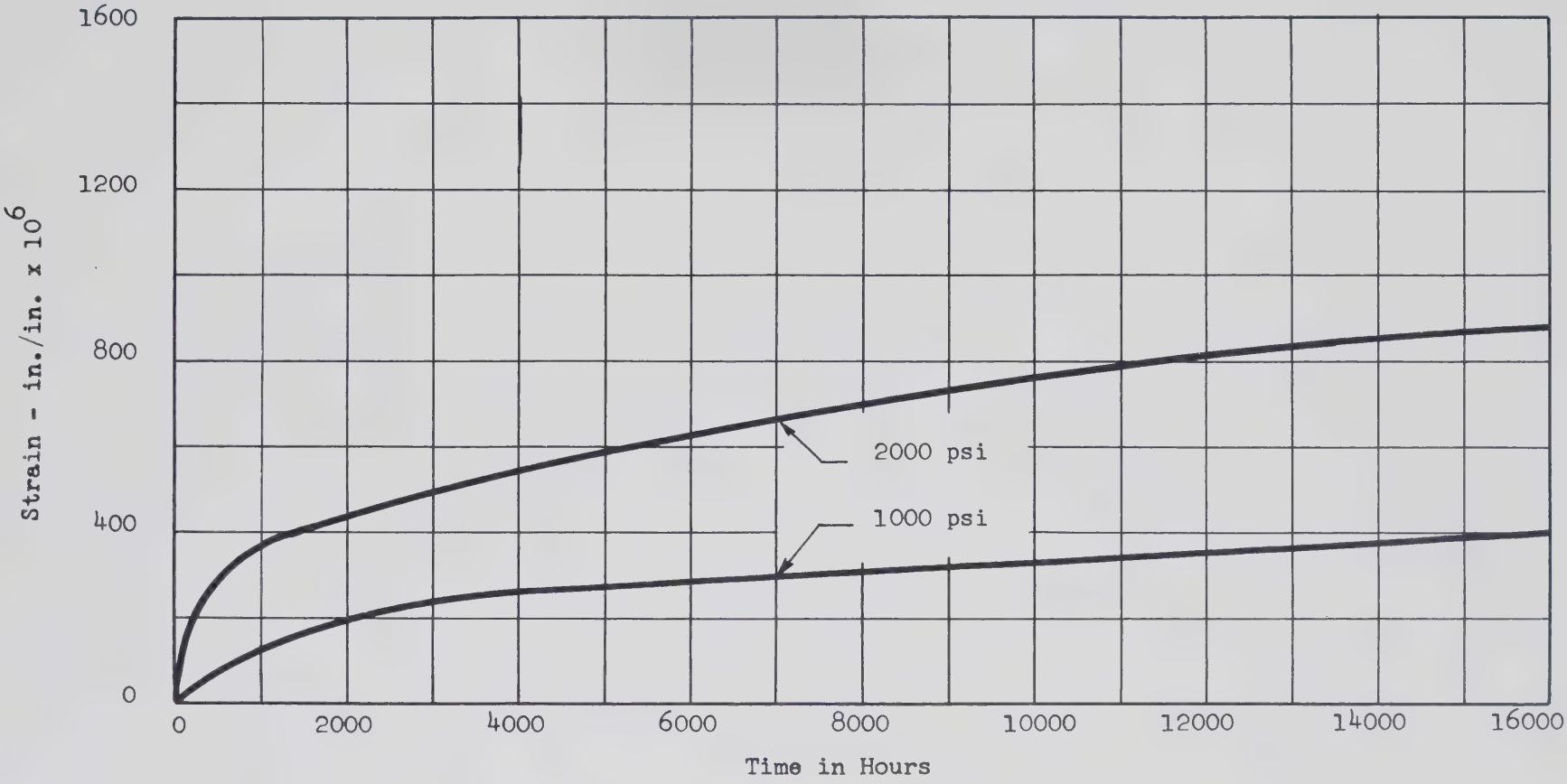


FIG. 13 CREEP STRAIN VERSUS TIME FOR 4 BY 16 IN. CYLINDERS OF GROUP G-3

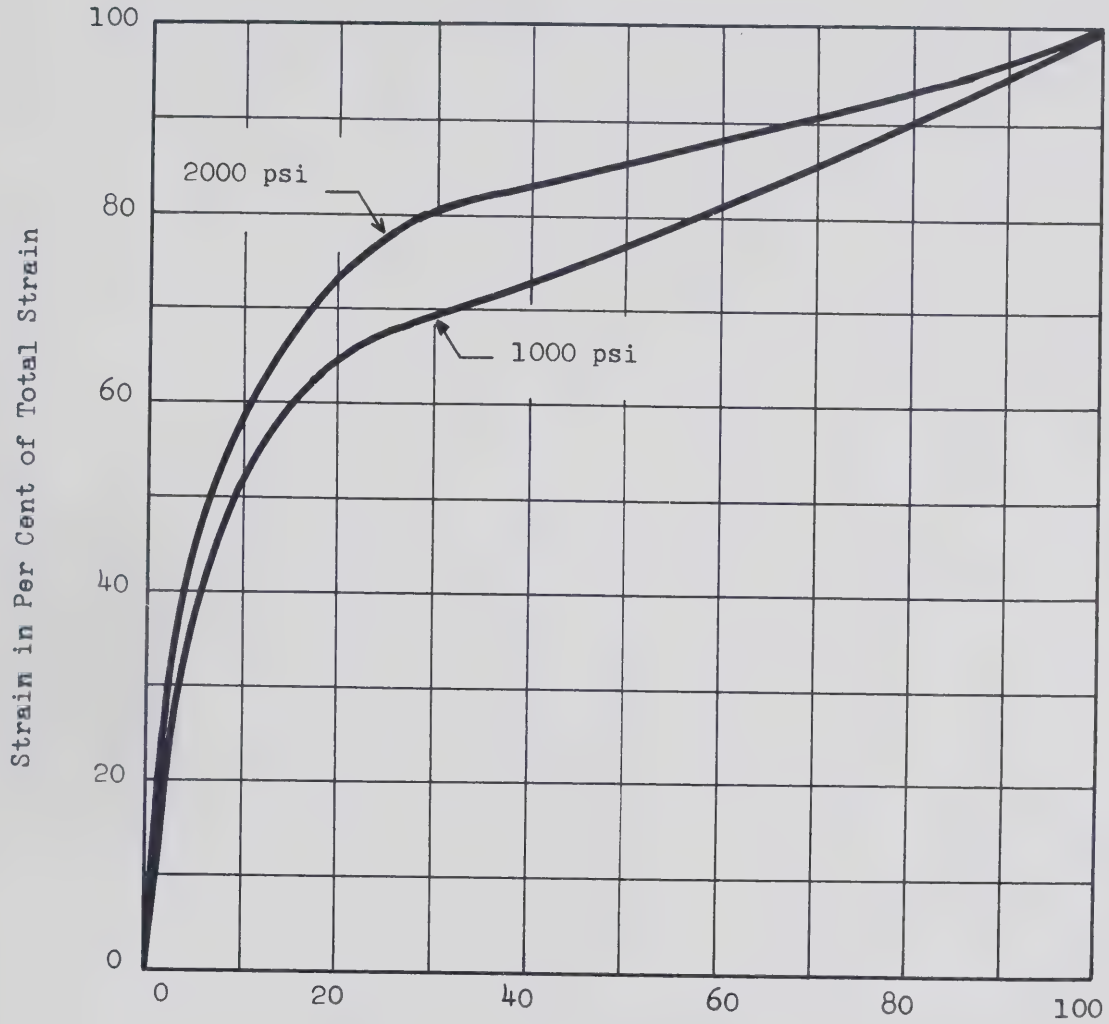


FIG. 14 DIMENSIONLESS PLOT OF CREEP STRAIN VERSUS TIME FOR CYLINDERS OF GROUP G-1

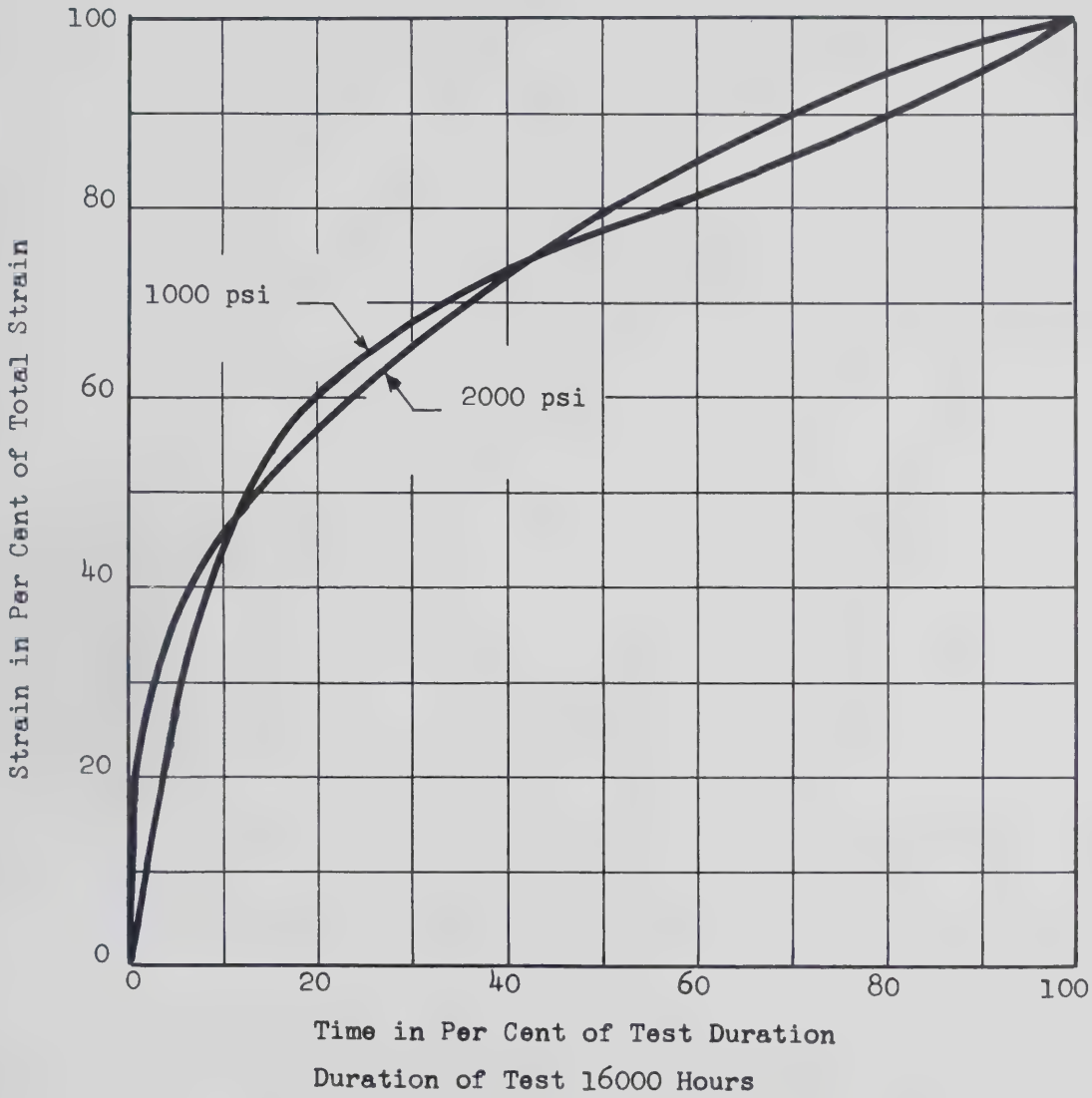


FIG. 15 DIMENSIONLESS PLOT OF CREEP STRAIN VERSUS TIME FOR CYLINDERS OF GROUP G-3

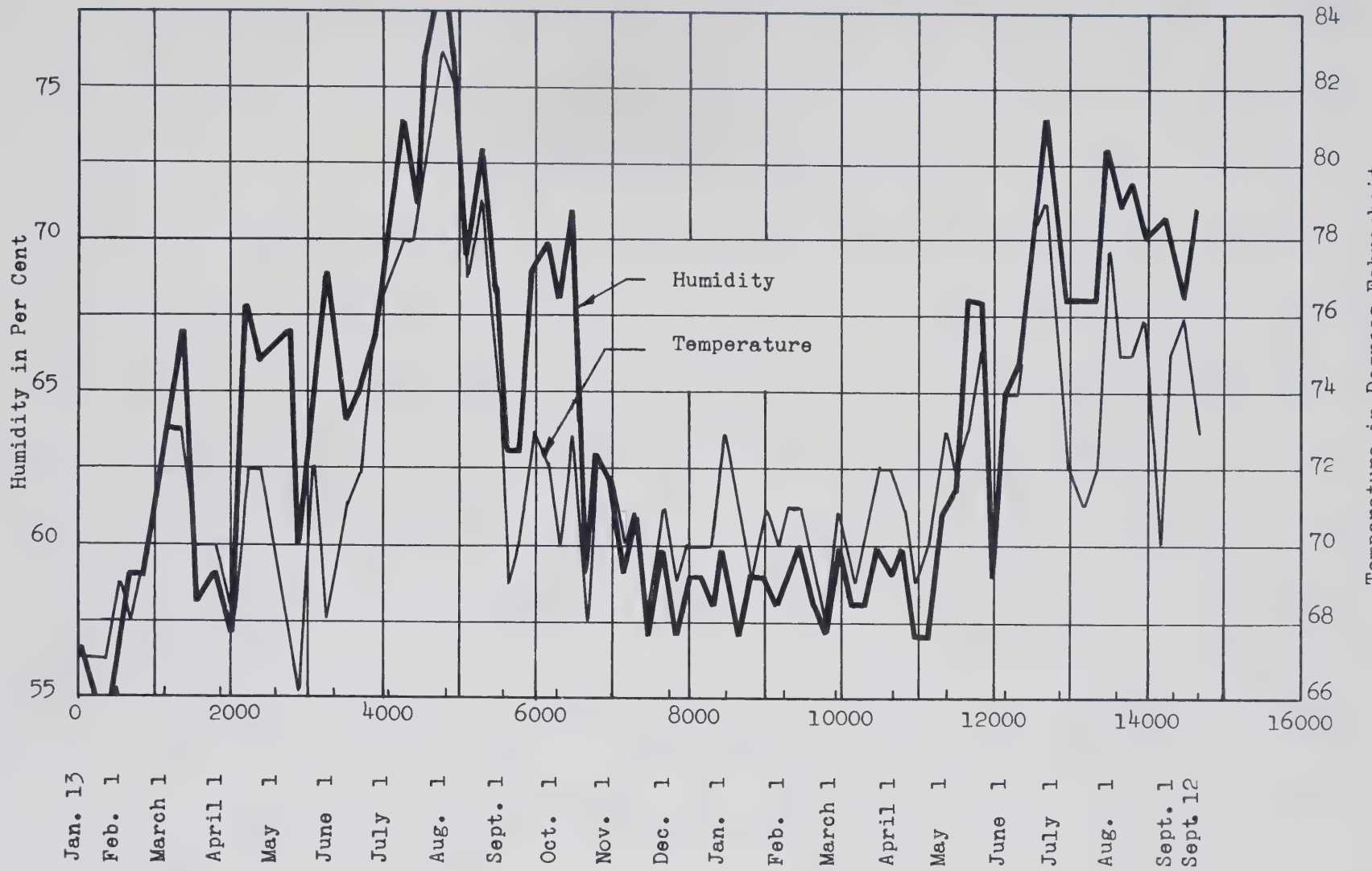


FIG. 16 TEMPERATURE AND HUMIDITY RECORD FOR THE PERIOD JANUARY 13, 1955 THROUGH SEPTEMBER 12, 1956

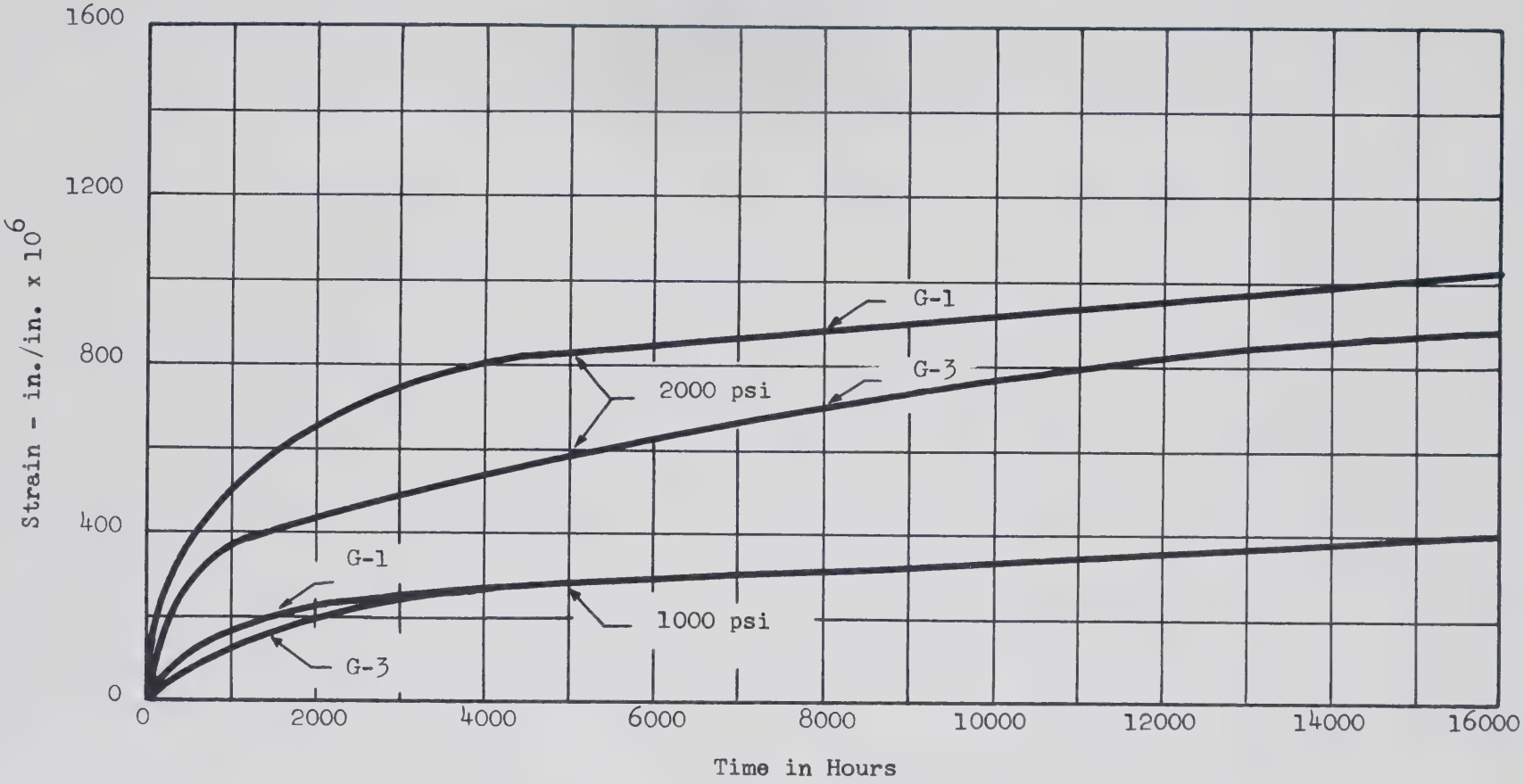


FIG. 17a COMPARISON OF CREEP STRAINS FOR CYLINDERS OF BEAMS G-1 AND G-3

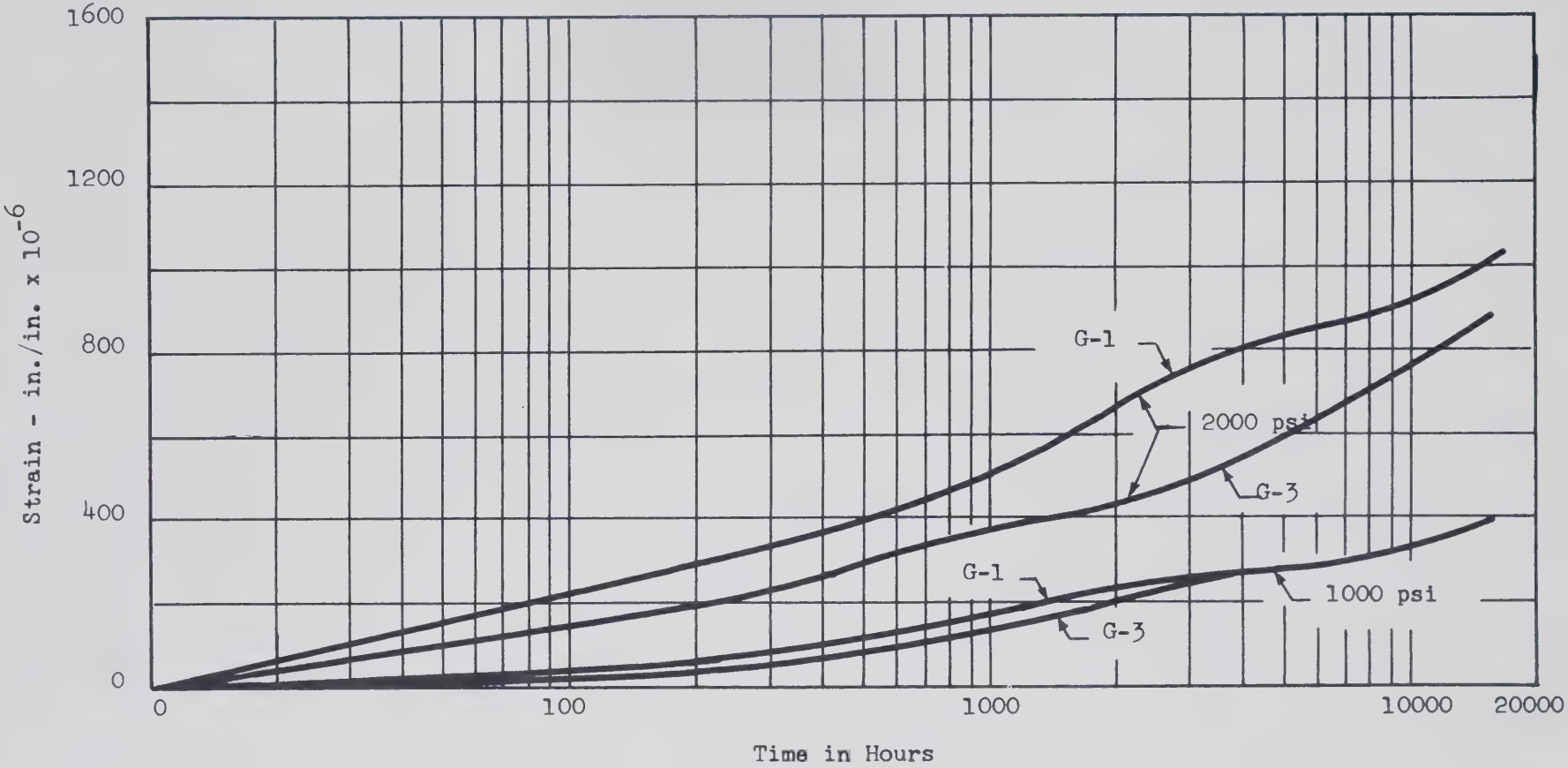


FIG. 17b COMPARISON OF CREEP STRAINS FOR CYLINDERS OF BEAMS G-1 AND G-3

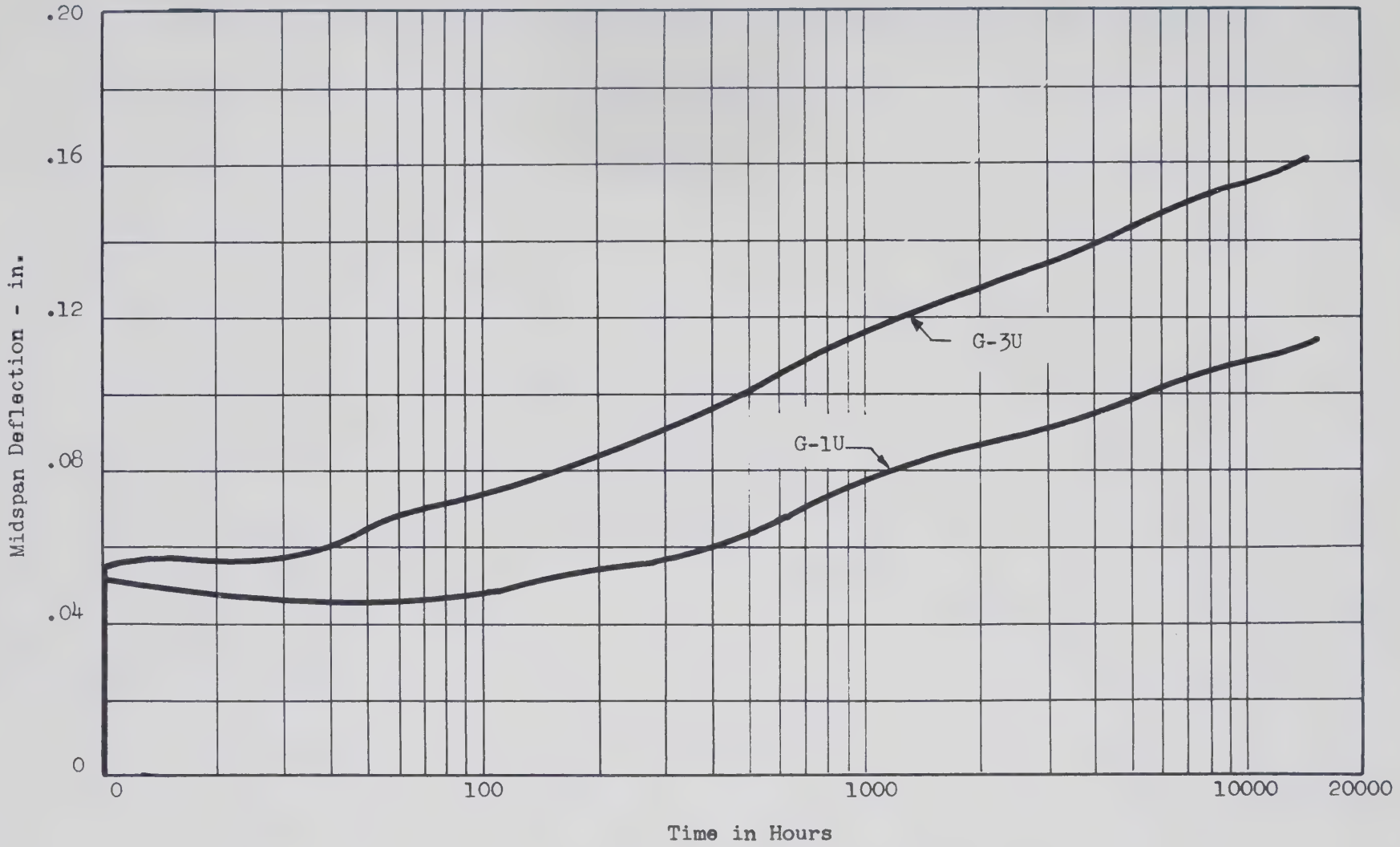


FIG. 18a COMPARISON OF MEASURED DEFLECTIONS FOR BEAMS G-1U AND G-3U

(Zero deflection corresponds to position of beam immediately before prestressing)

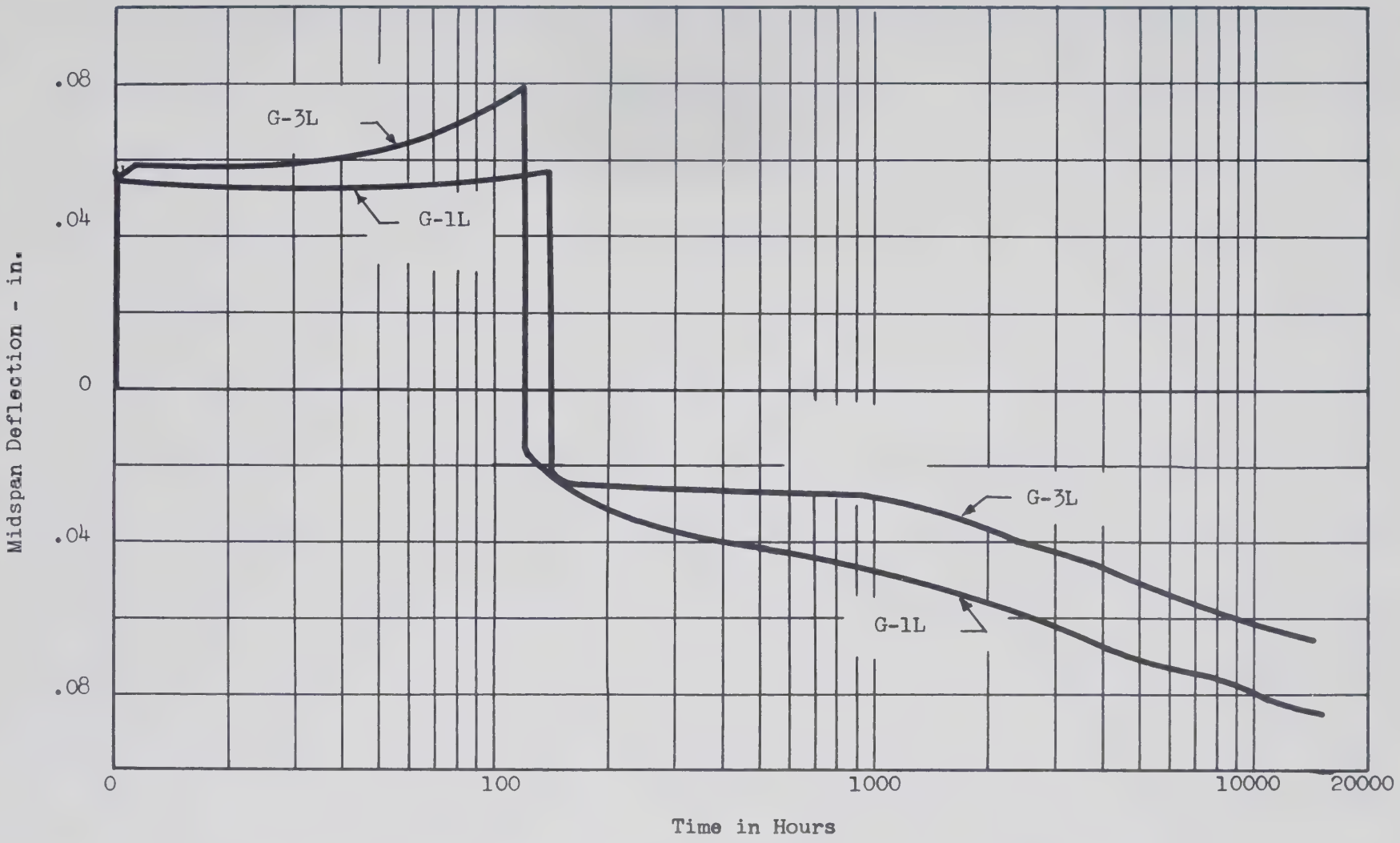


FIG. 18b COMPARISON OF MEASURED DEFLECTIONS FOR BEAMS G-1 AND G-3L

(Zero deflection corresponds to position of beam immediately before prestressing)

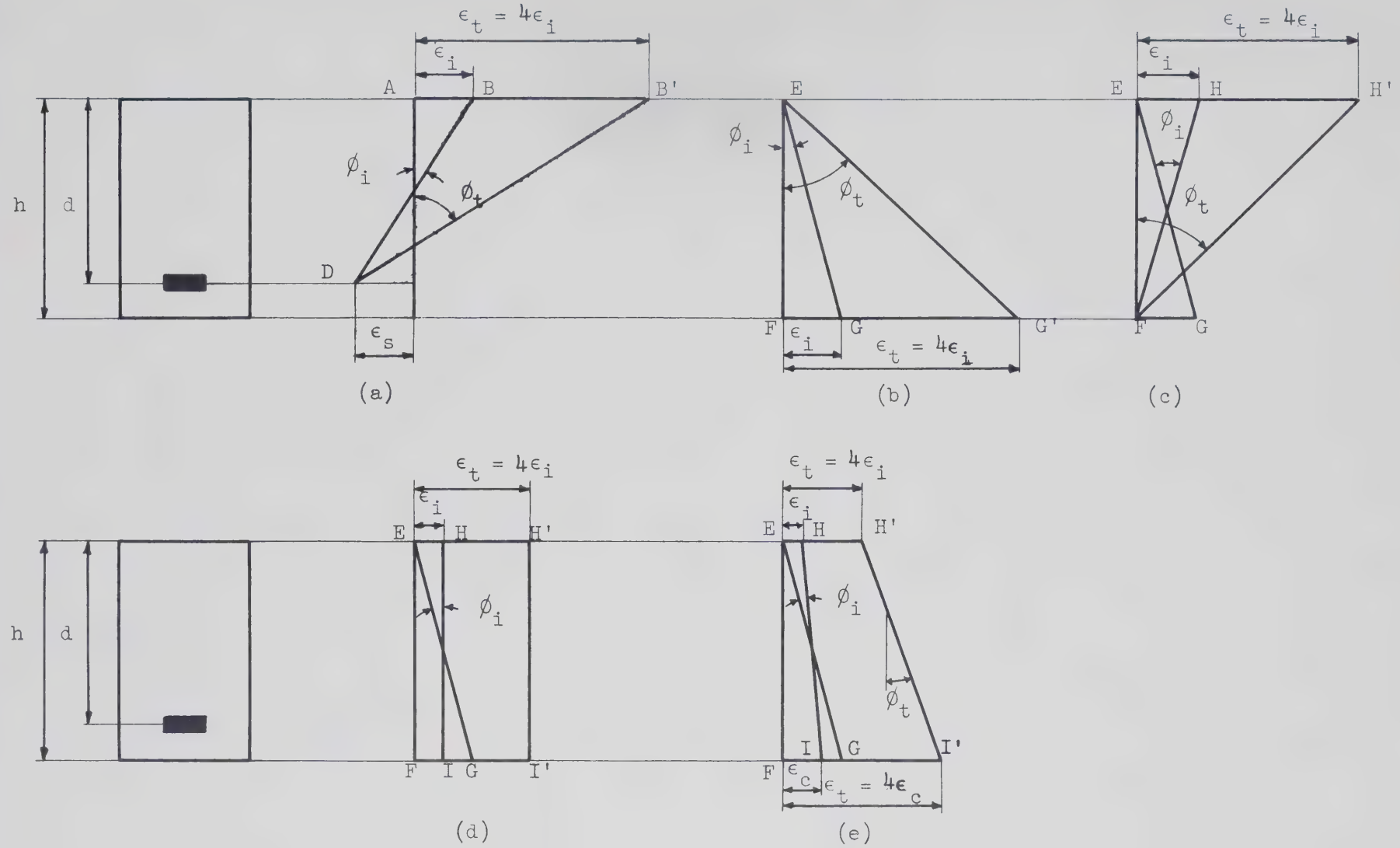
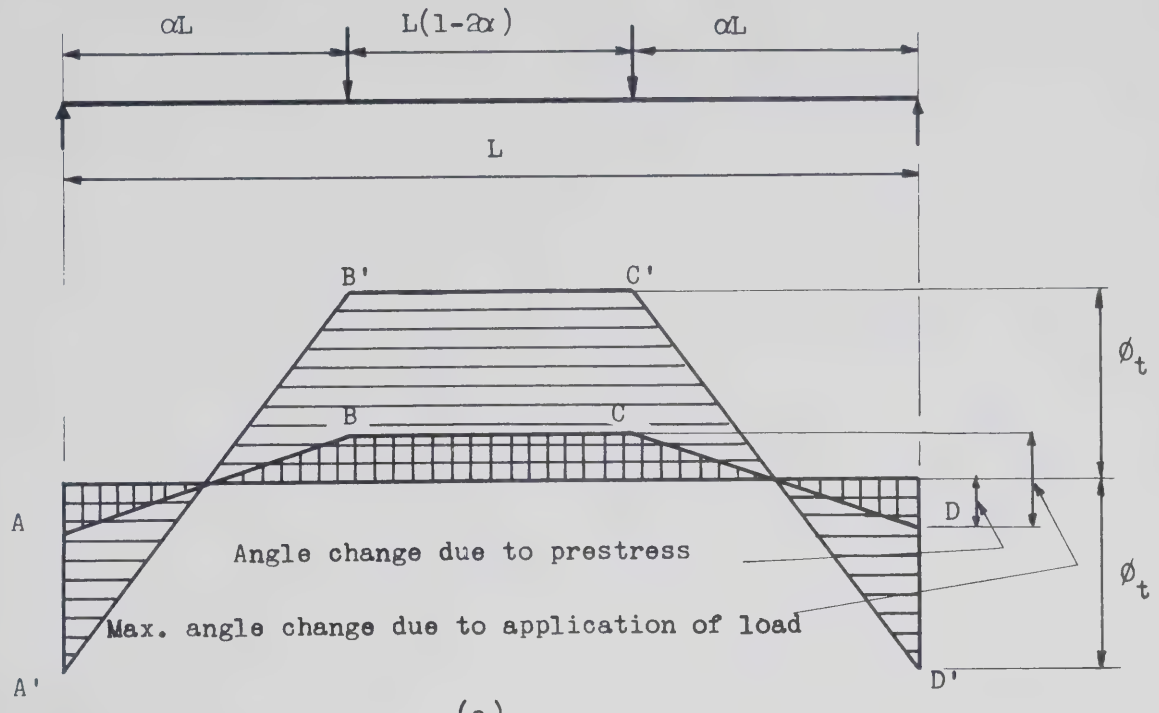
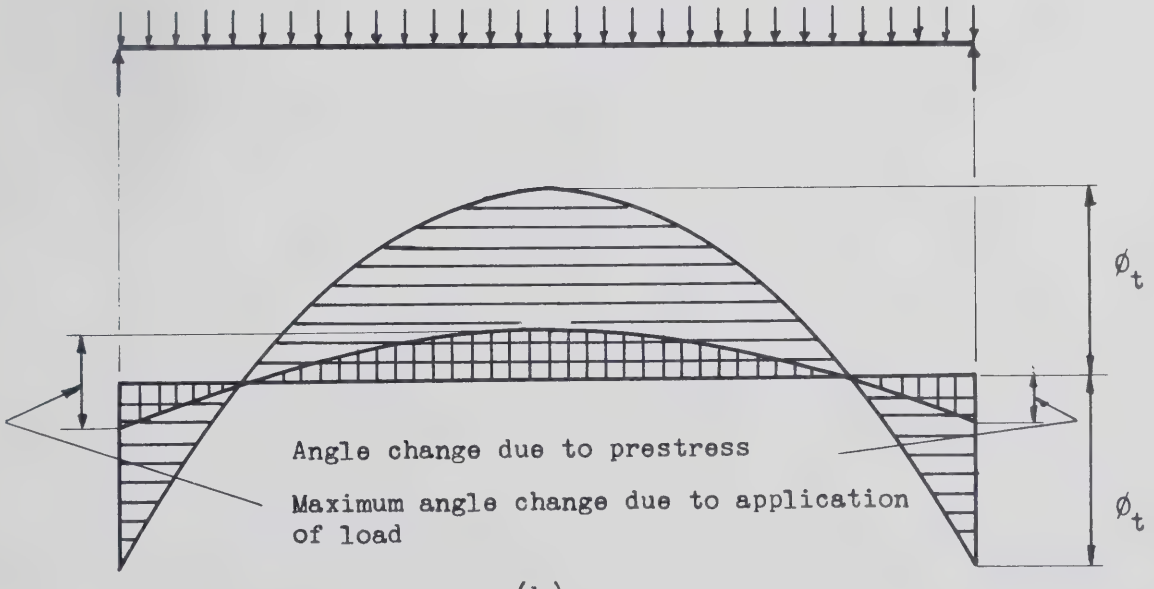


FIG. 19 COMPARISON OF LONG-TIME AND INSTANTANEOUS ANGLE CHANGES CORRESPONDING TO VARIOUS COMBINATIONS OF LOAD AND PRESTRESS



(a)



(b)

FIG. 20 DISTRIBUTION OF LONG-TIME AND INSTANTANEOUS ANGLE CHANGES ALONG THE SPAN OF SIMPLY-SUPPORTED BEAMS WITH STRAIGHT TENDONS

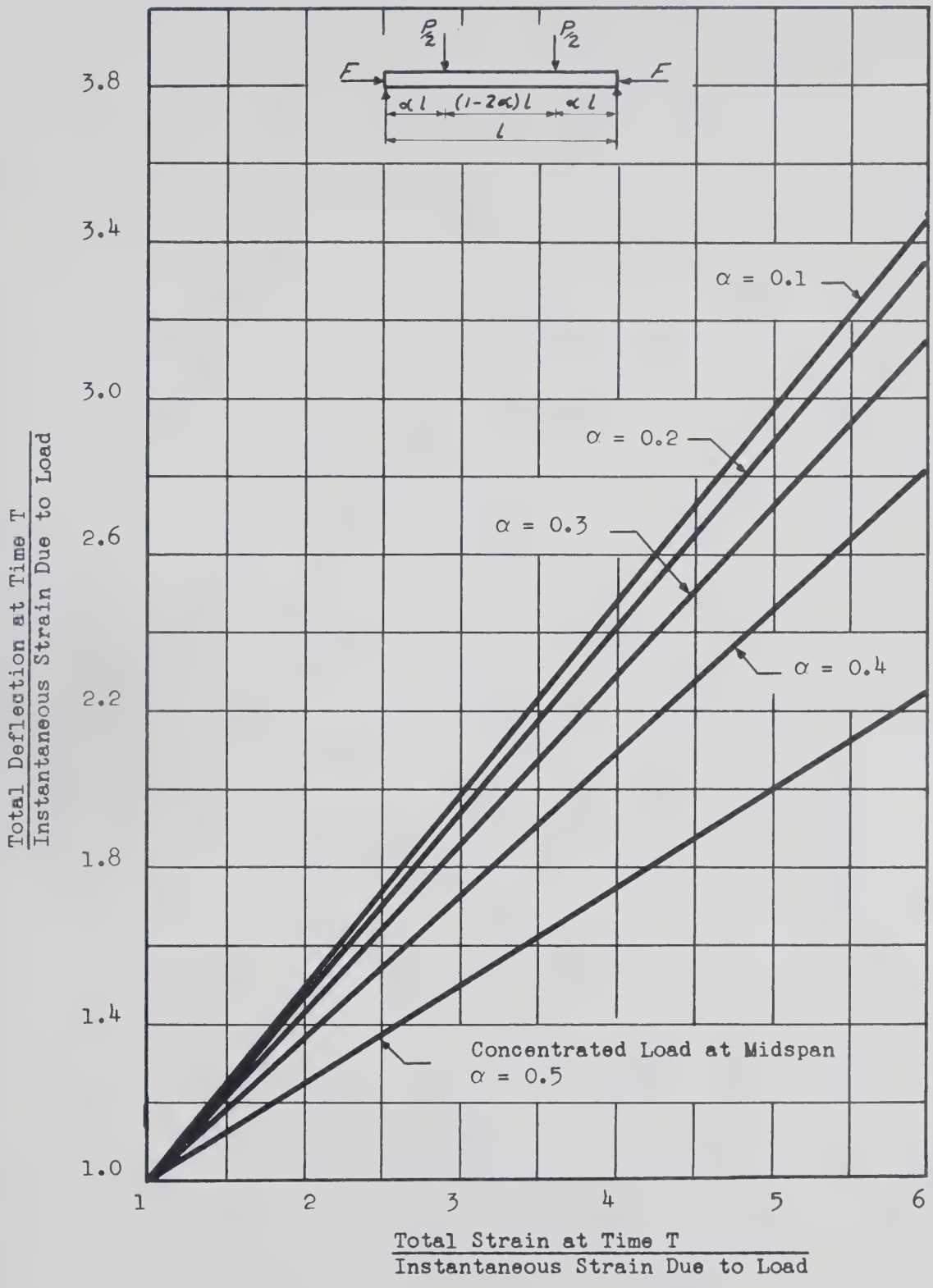


FIG. 21 DIMENSIONLESS PLOT OF DEFLECTION VERSUS STRAIN

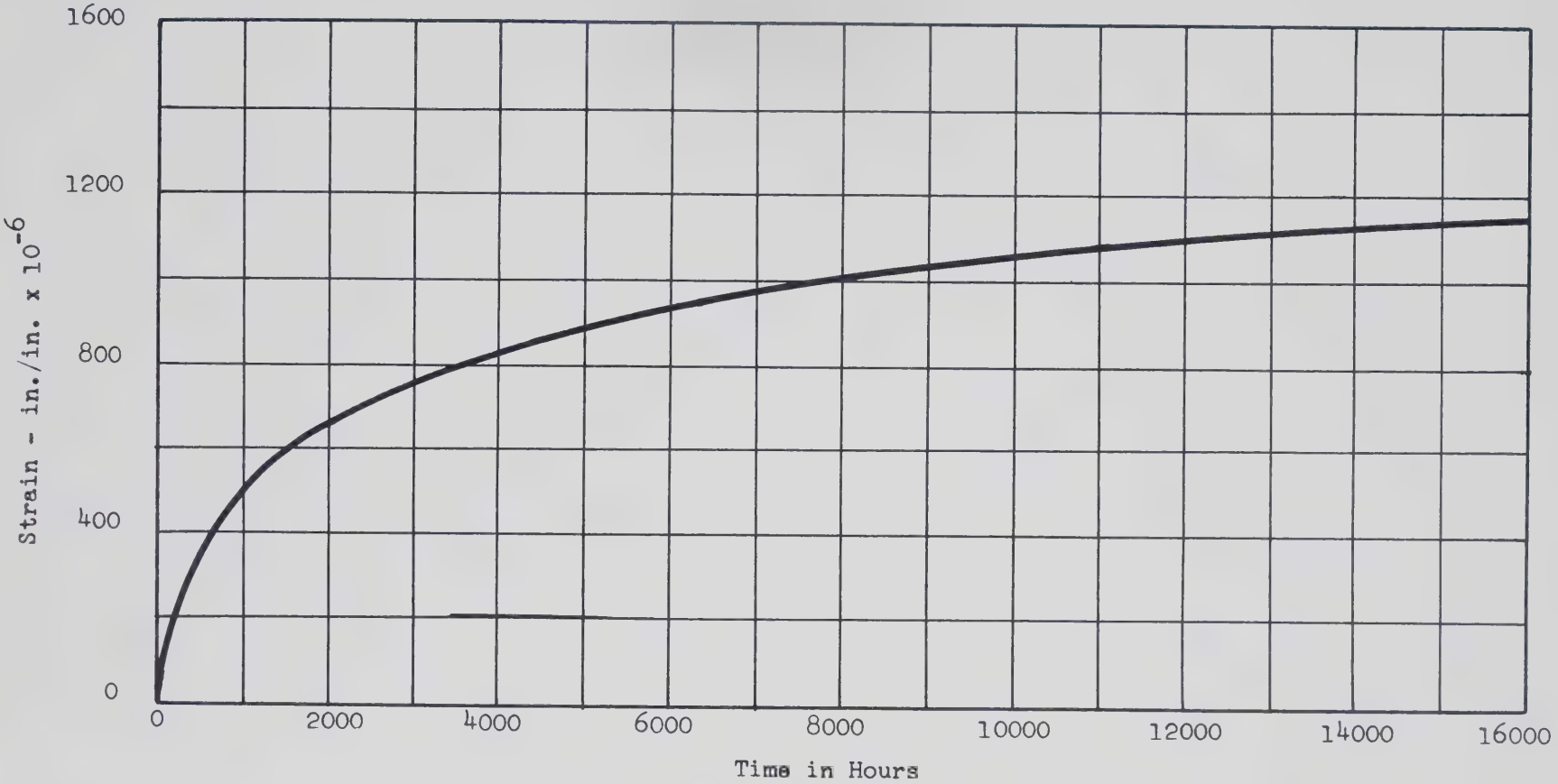


FIG. 22 CREEP STRAIN VERSUS TIME AT 2000 psi DERIVED FROM STRAIN MEASUREMENTS ON BEAM G-3U

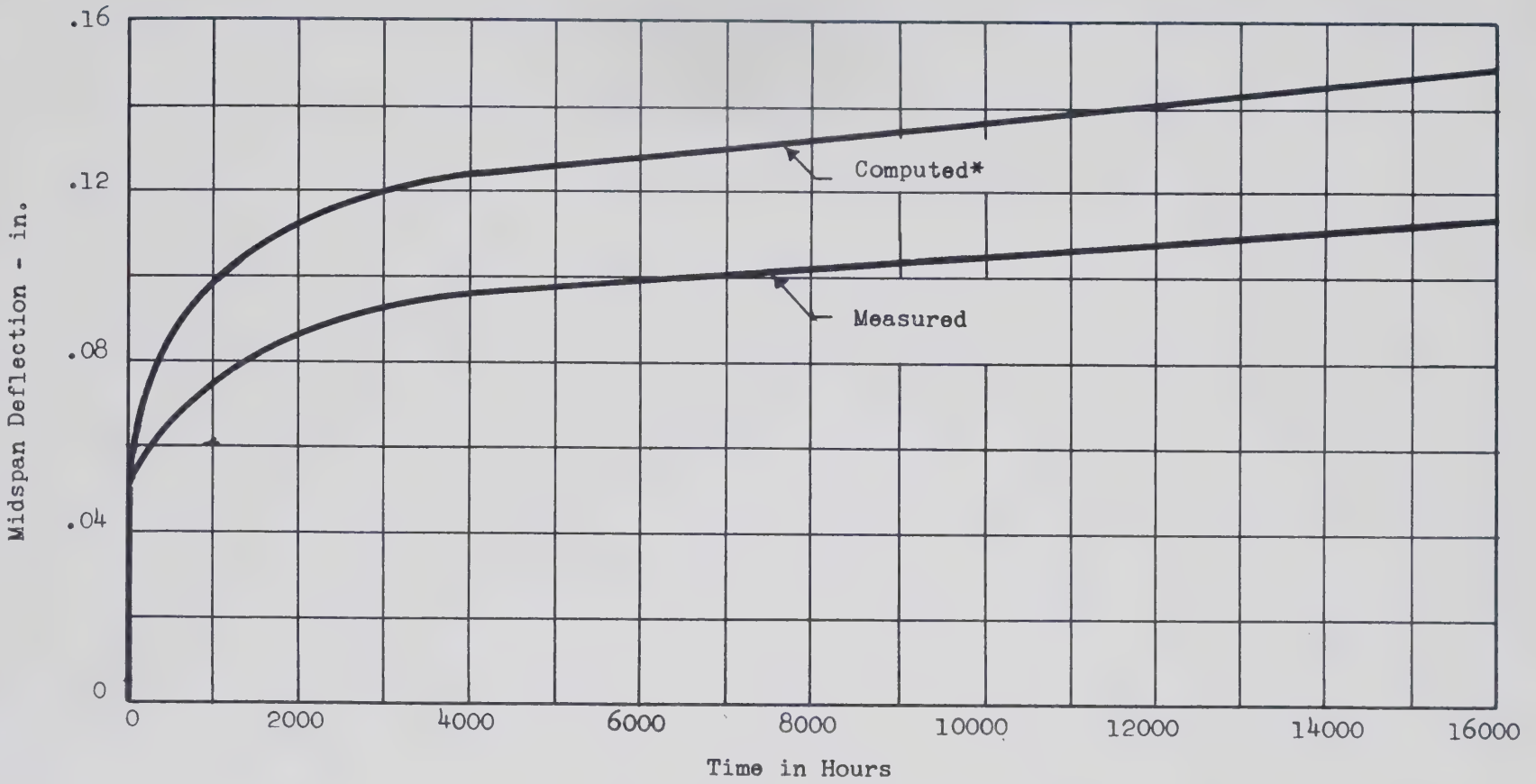


FIG. 23 COMPARISON OF MEASURED AND COMPUTED DEFLECTION FOR BEAM G-1U

*Computation based on creep strains measured from cylinders of group G-1.

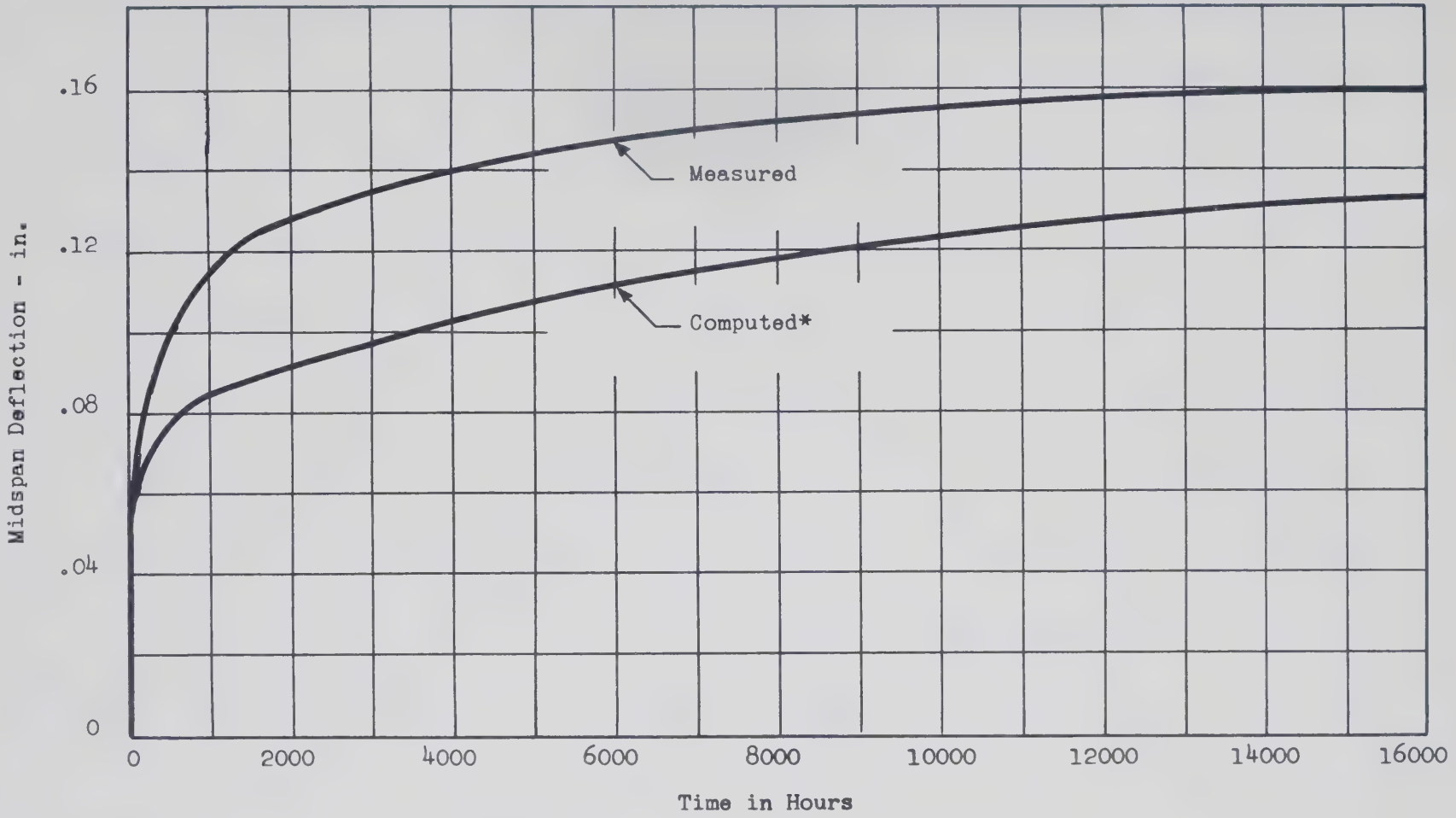


FIG. 24 COMPARISON OF MEASURED AND COMPUTED DEFLECTIONS FOR BEAM G-3U

*Computation based on creep strains measured from cylinders of beams G-3.

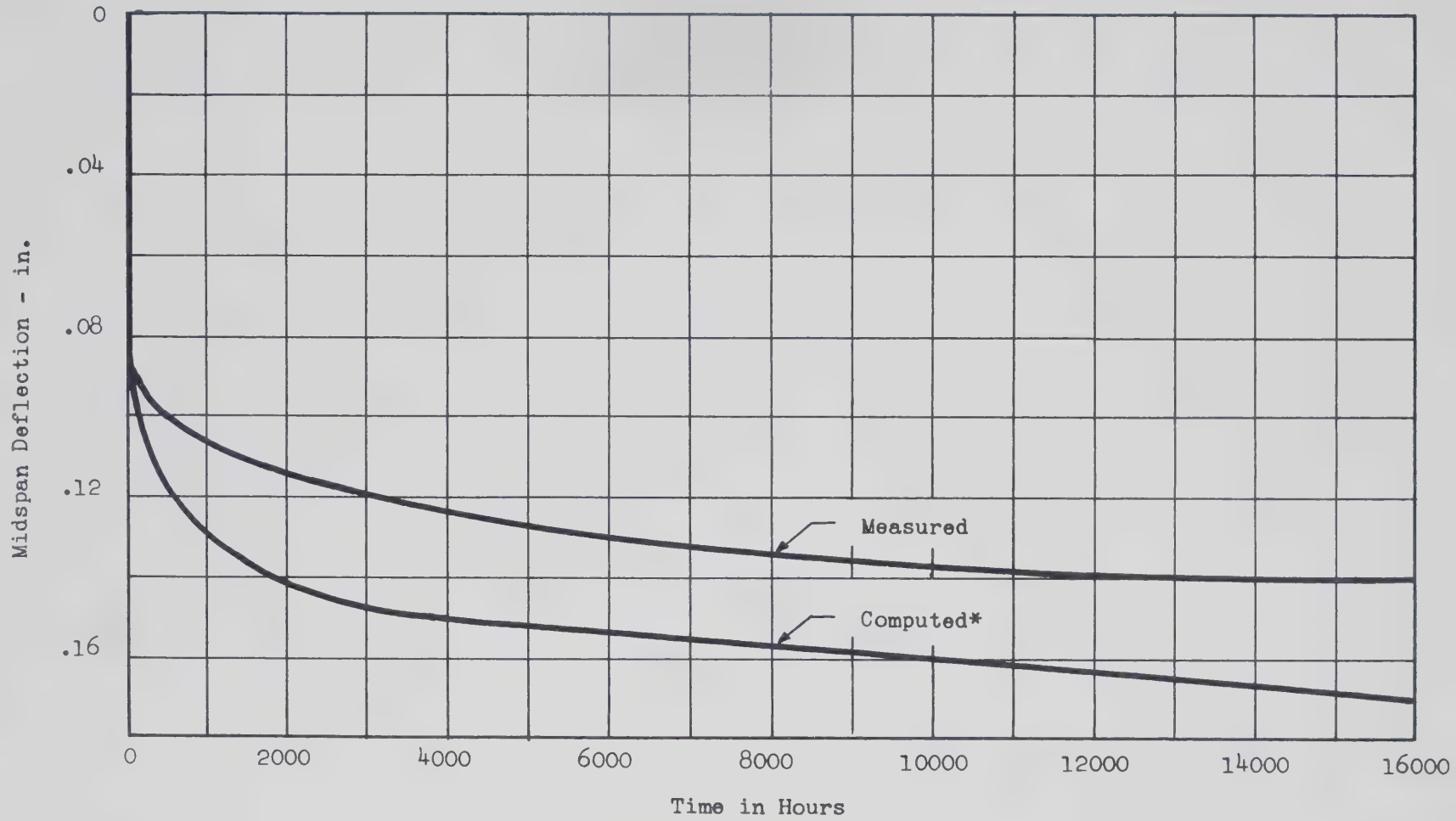


FIG. 25 COMPARISON OF MEASURED AND COMPUTED DEFLECTIONS FOR BEAM G-1L

*Computation based on creep strains measured from cylinders of beams G-1.

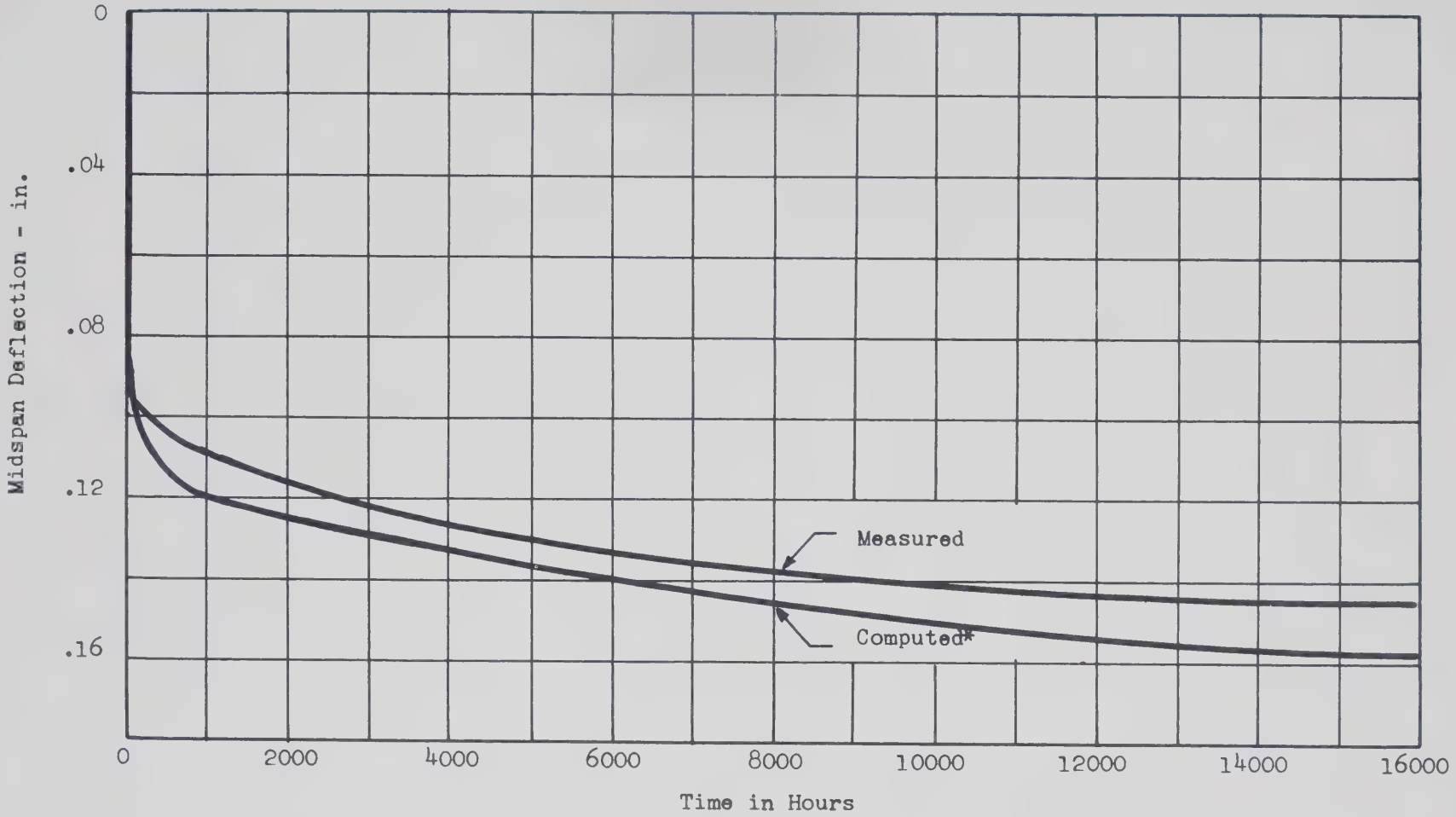


FIG. 26 COMPARISON OF MEASURED AND COMPUTED DEFLECTIONS FOR BEAM G-3L

*Computation based on creep strains measured from cylinders of beams G-3.

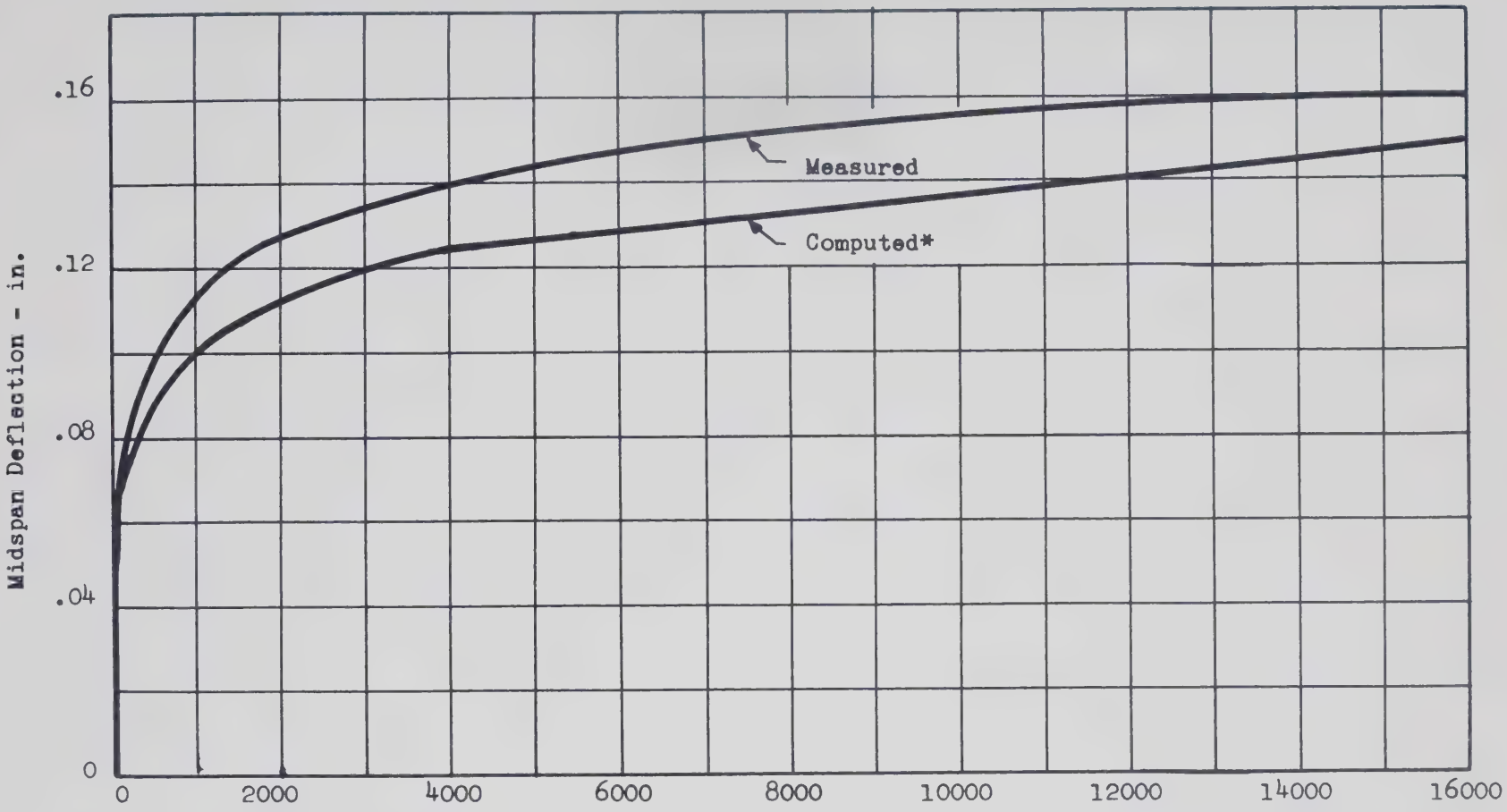


FIG. 27 COMPARISON OF MEASURED AND COMPUTED DEFLECTIONS FOR BEAM G-3U

*Computation based on creep strains measured from cylinders of beams G-1

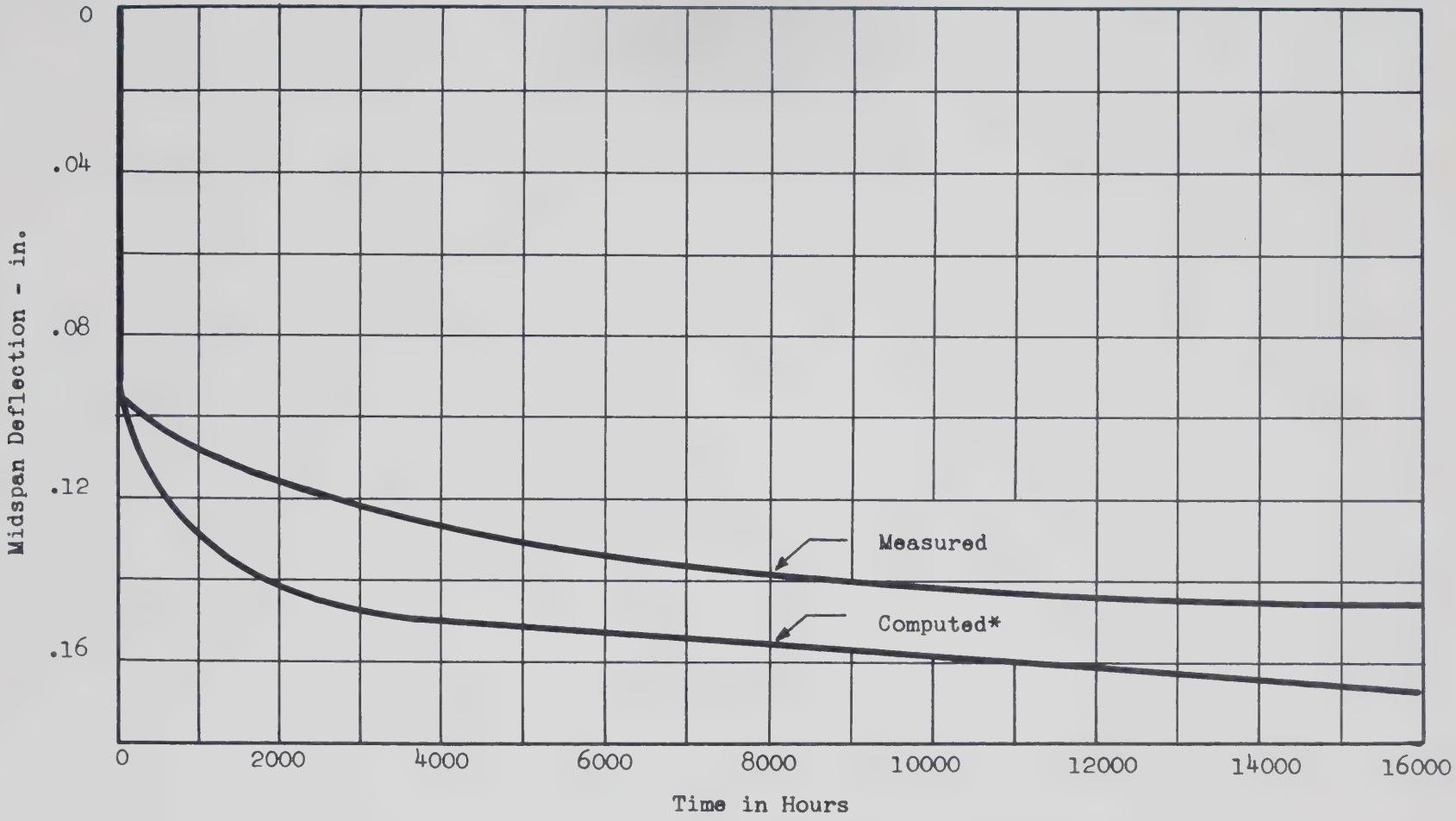


FIG. 28 COMPARISON OF MEASURED AND COMPUTED DEFLECTIONS FOR BEAM G-3L

*Computation based on creep strains measured from cylinders of beams G-1.

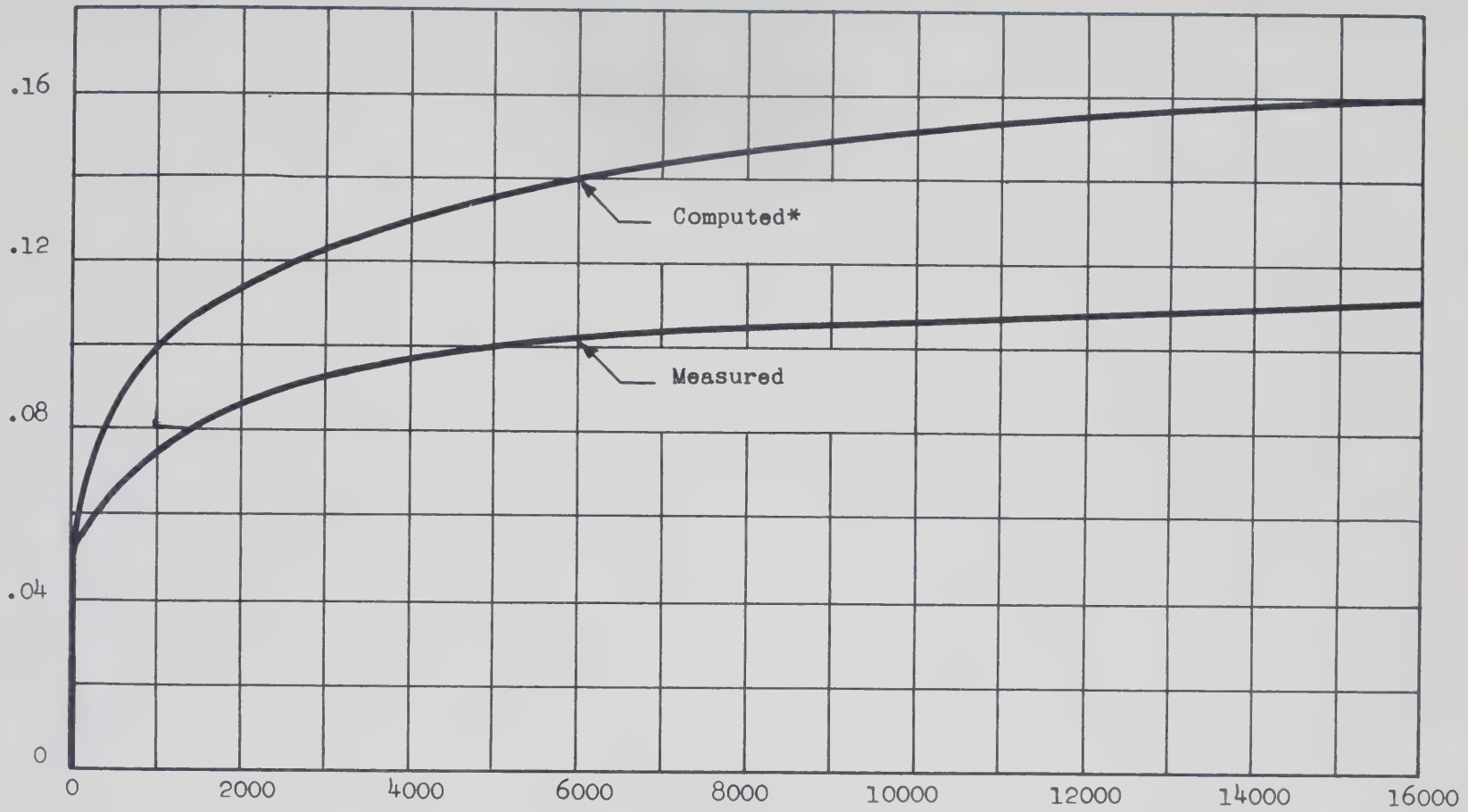


FIG. 29 COMPARISON OF MEASURED AND COMPUTED DEFLECTIONS FOR BEAM G-1U

*Computation based on creep function derived from measured strains on beam G-3U.

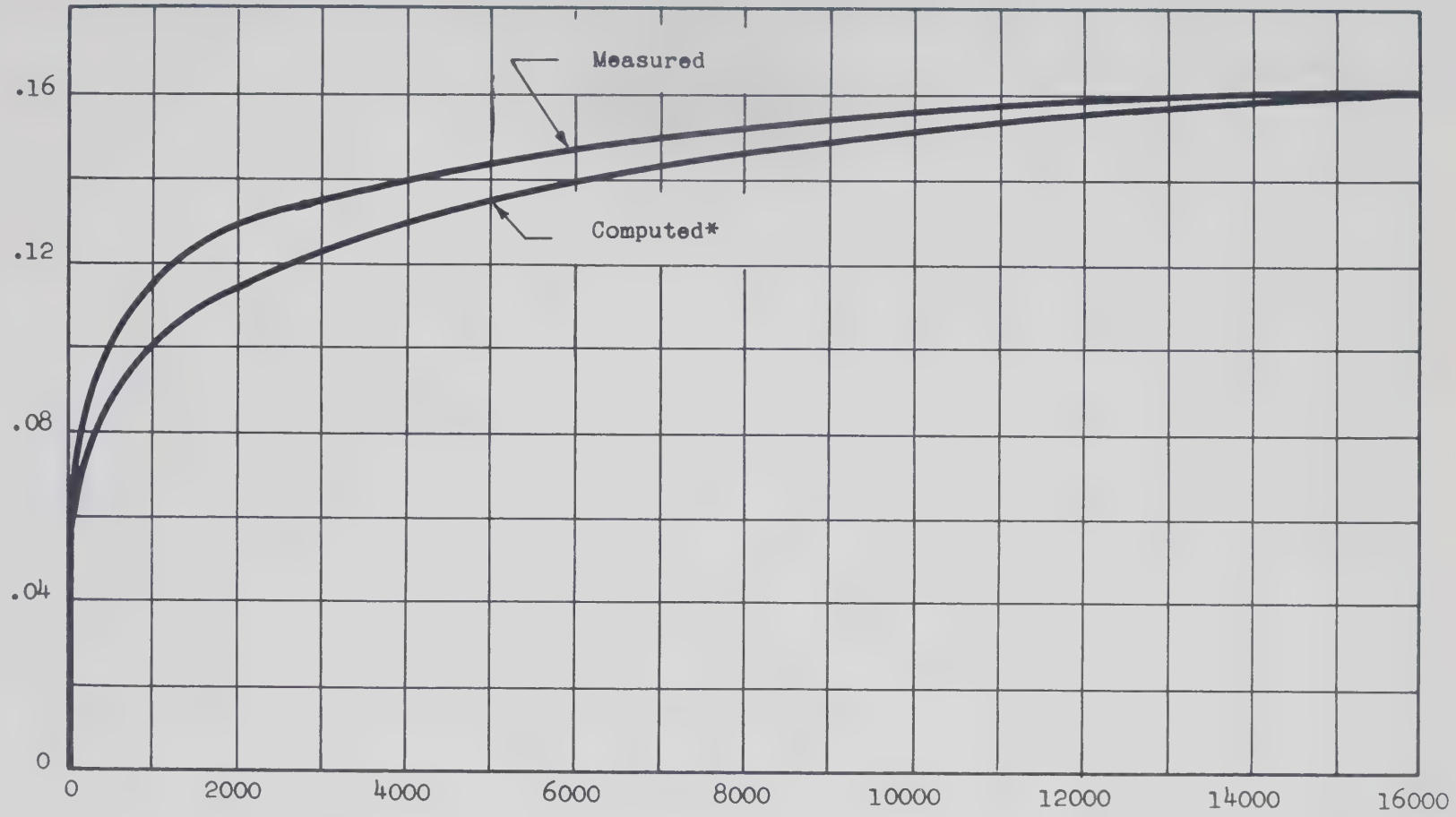


FIG. 30 COMPARISON OF MEASURED AND COMPUTED DEFLECTIONS FOR BEAM G-3U

*Computation based on creep function derived from measured strains on beam G-3U.

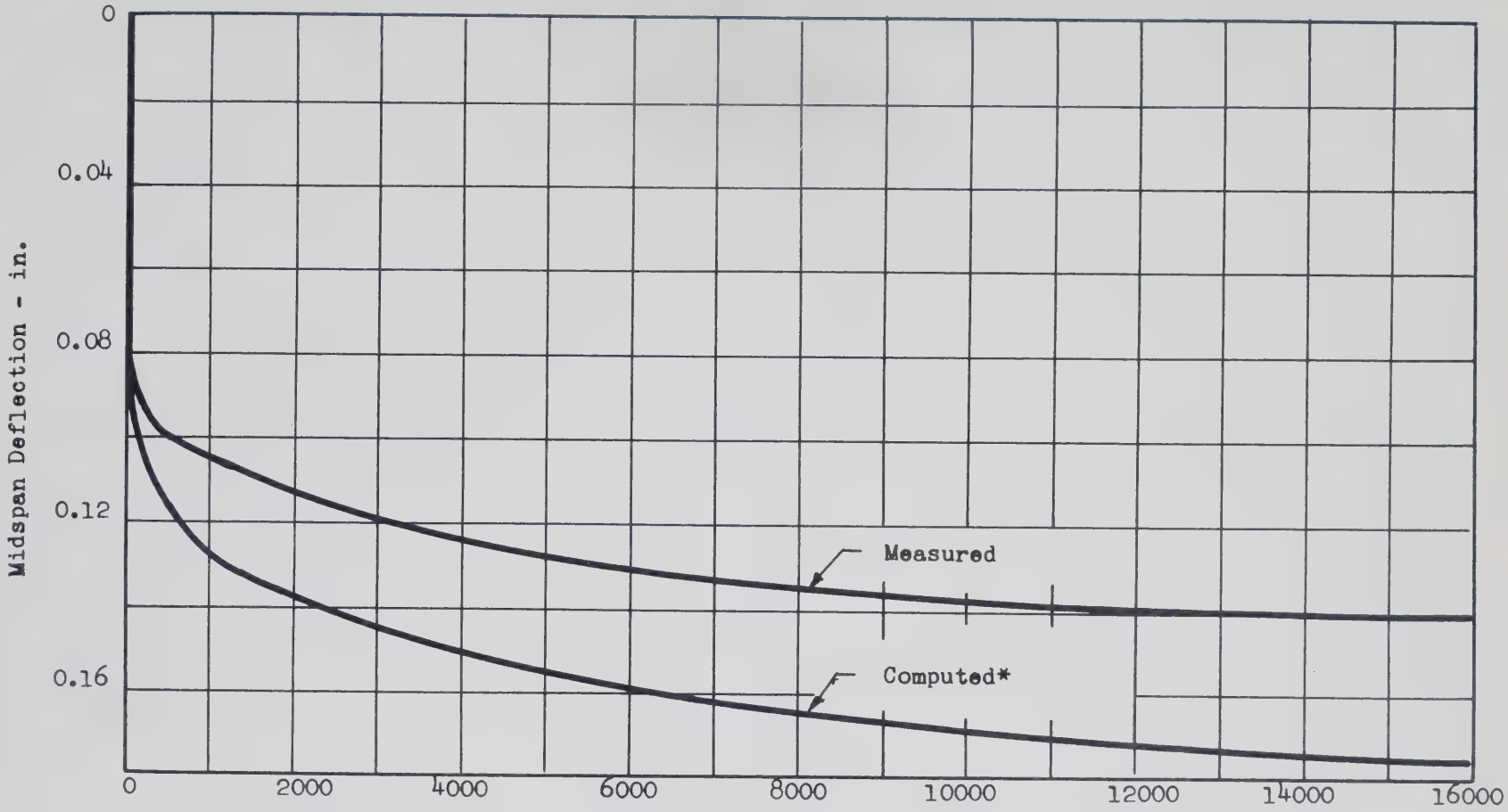


FIG. 31 COMPARISON OF MEASURED AND COMPUTED DEFLECTIONS FOR BEAM G-1L

*Computations based on creep function derived from measured strains on beam G-3U.

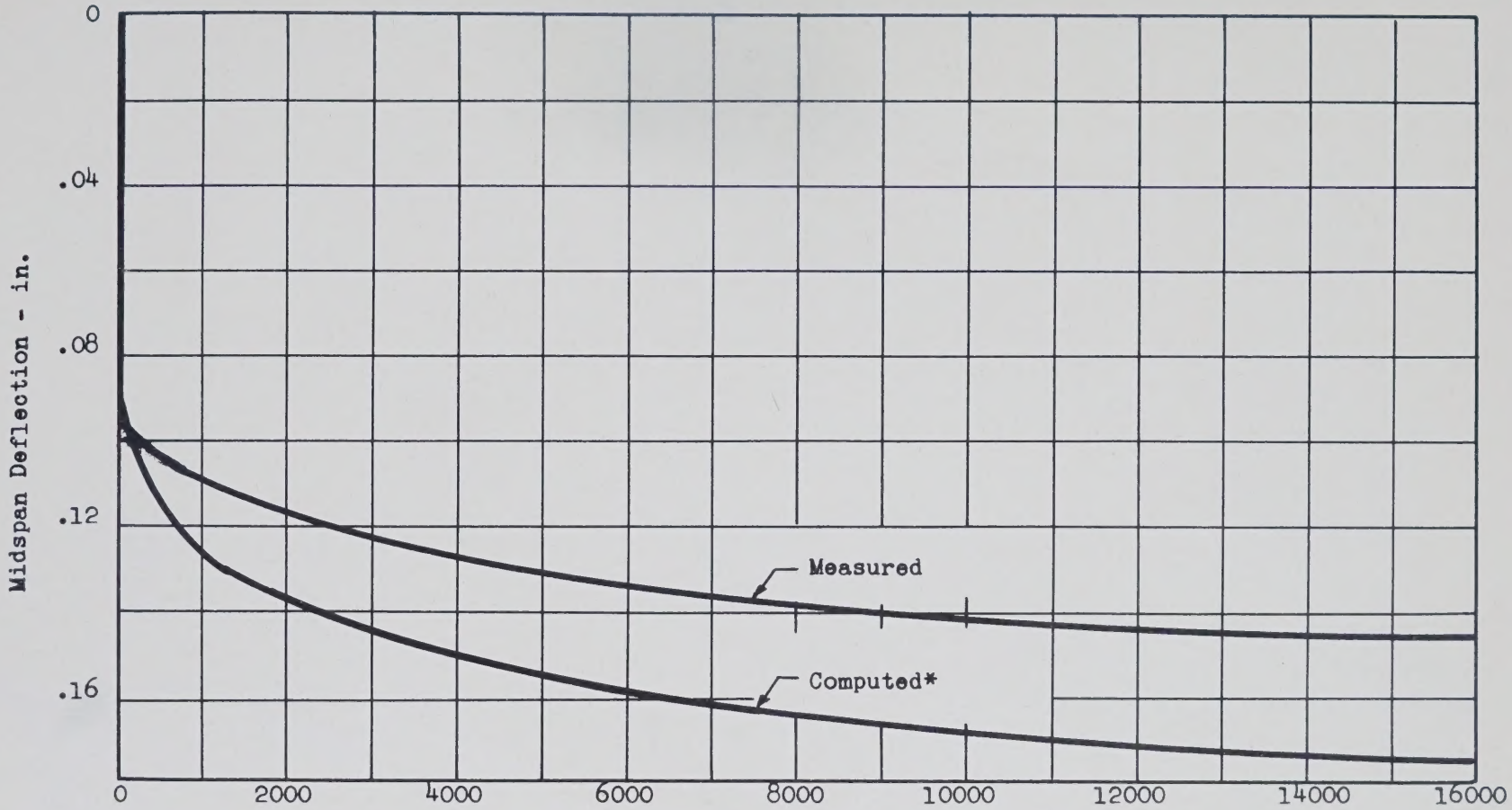


FIG. 32 COMPARISON OF MEASURED AND COMPUTED DEFLECTION FOR BEAM G-3L

*Computation based on creep function derived from measured strains on beam G-3U.

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